old title: A novel modeling approach for diffusion-dominated systems

Principal Component Analysis (PCA) and Transient Seasonal Deformation: A new model for seasonally correlated hydrologic deformation

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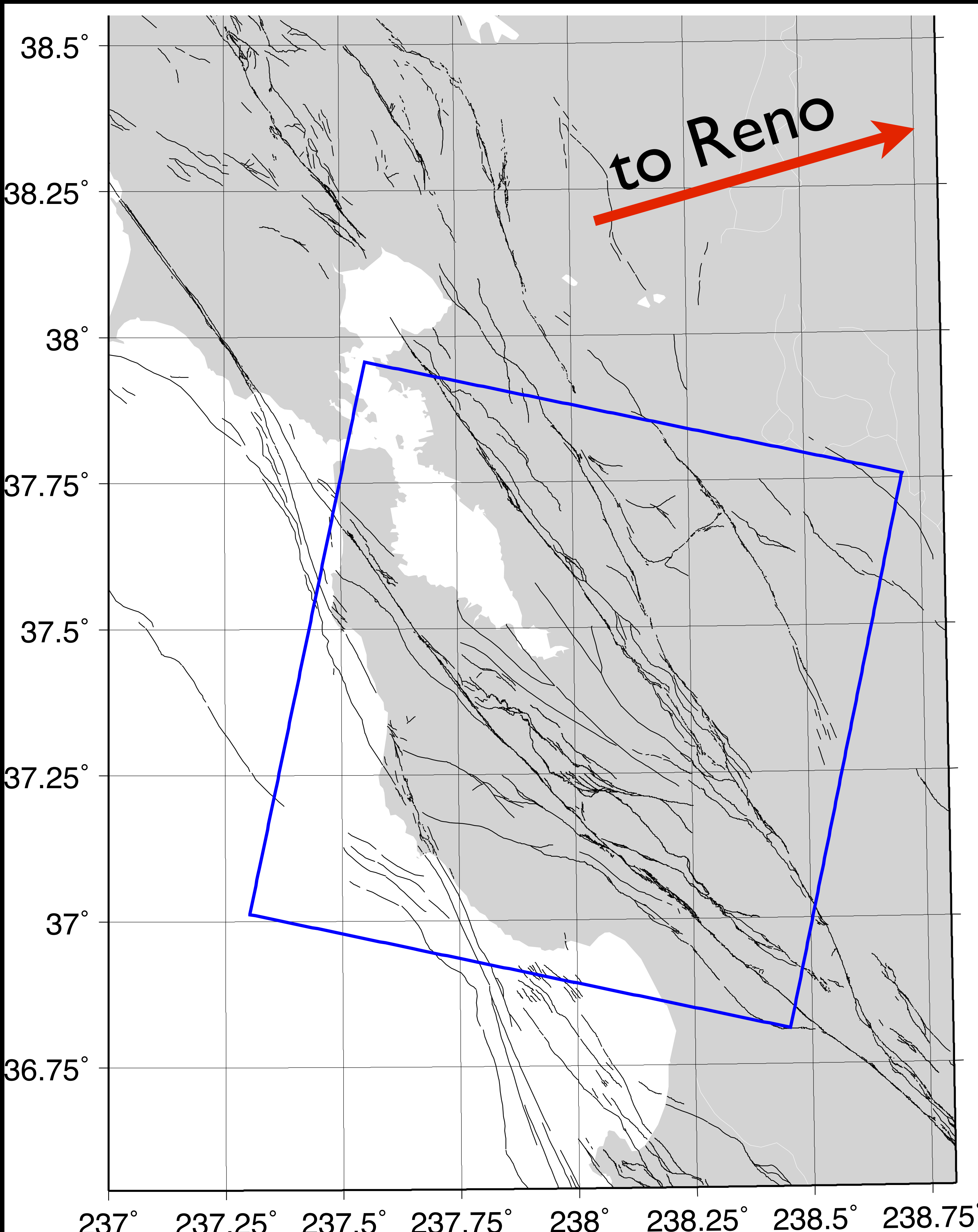


Data

- PS-InSAR from ERS-1 and ERS-2
- 50 Aquisitions between 1995-09 and 2001-01
- O{10^5} spatial points

Lipovsky et al., *PCA and seasonal deformation*

Method: Principal Component Analysis

- Goal: $\mathbf{D}(\mathrm{x,t}) \longrightarrow \mathbf{X}(\mathbf{x})\ \mathbf{T}(\mathrm{t})$

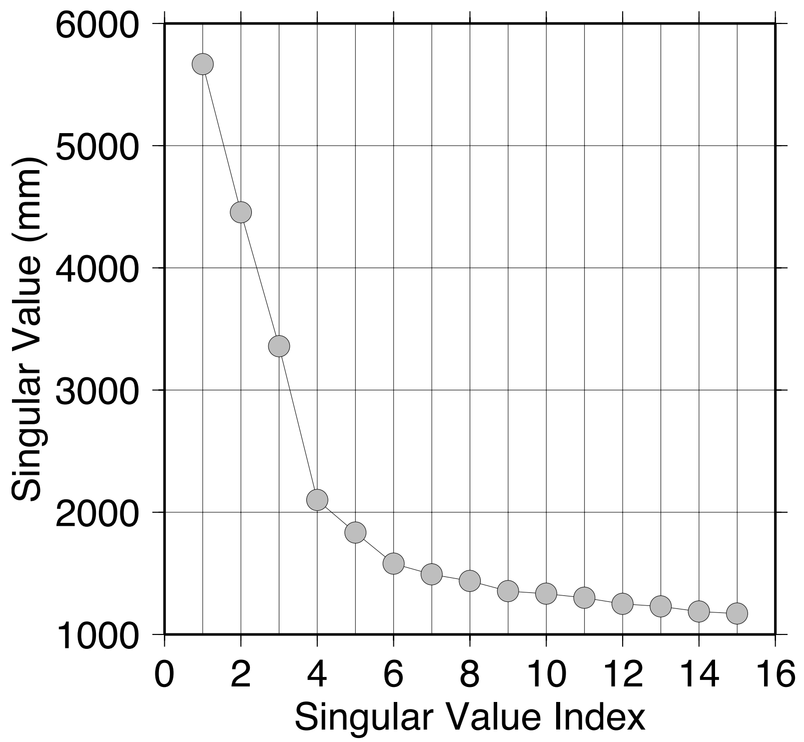
- PCA is the generalized eigenvalue problem,

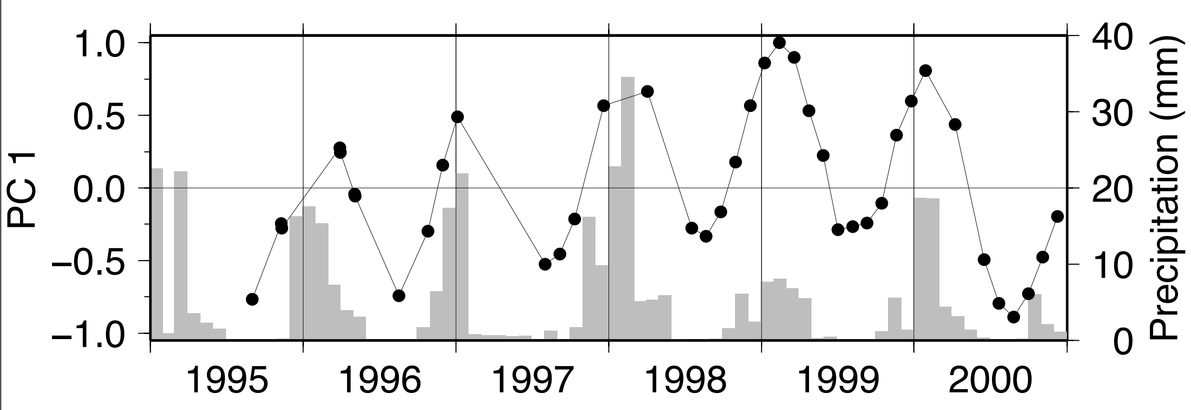
$$\mathbf{D}(\mathbf{x},\mathrm{t}) = \mathbf{U}\ \mathbf{L}\ \mathbf{T}',$$
$$\mathbf{X} = \mathbf{U}\ \mathbf{L}$$

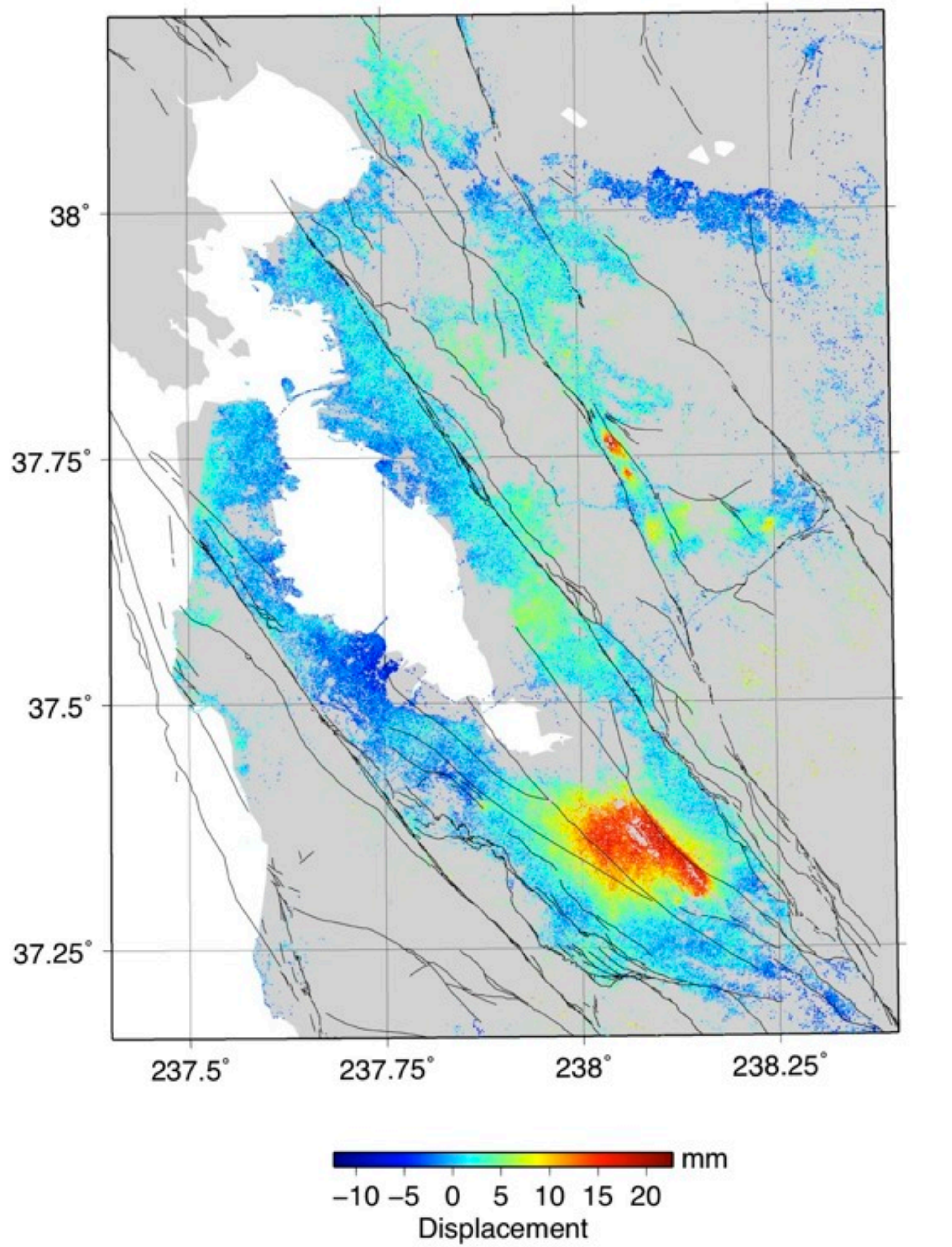
- When sorted by singular value, the patterns are ordered by the *percent of data variance* contained in each pattern

Lipovsky et al., *PCA and seasonal deformation*

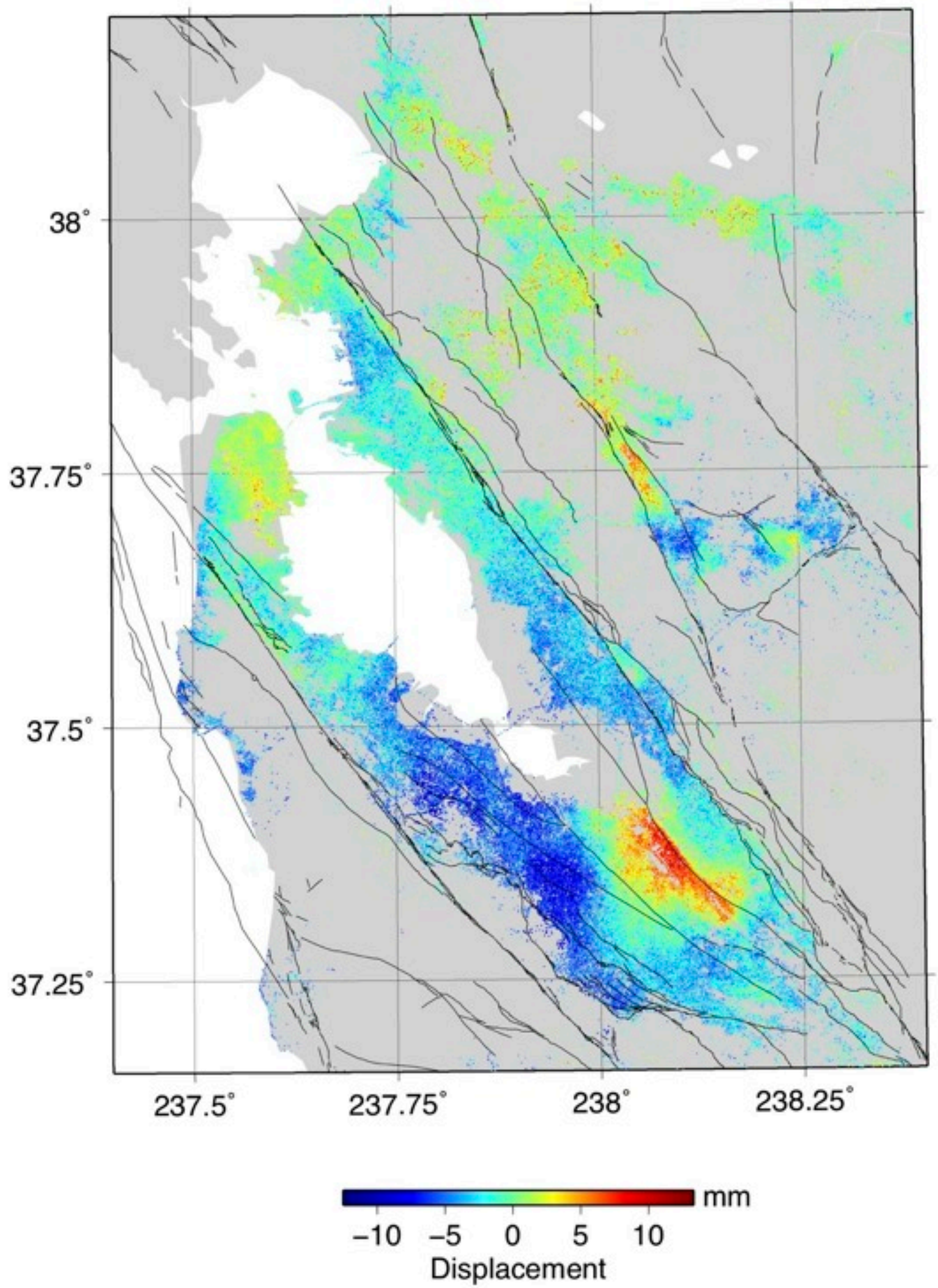
PC 1

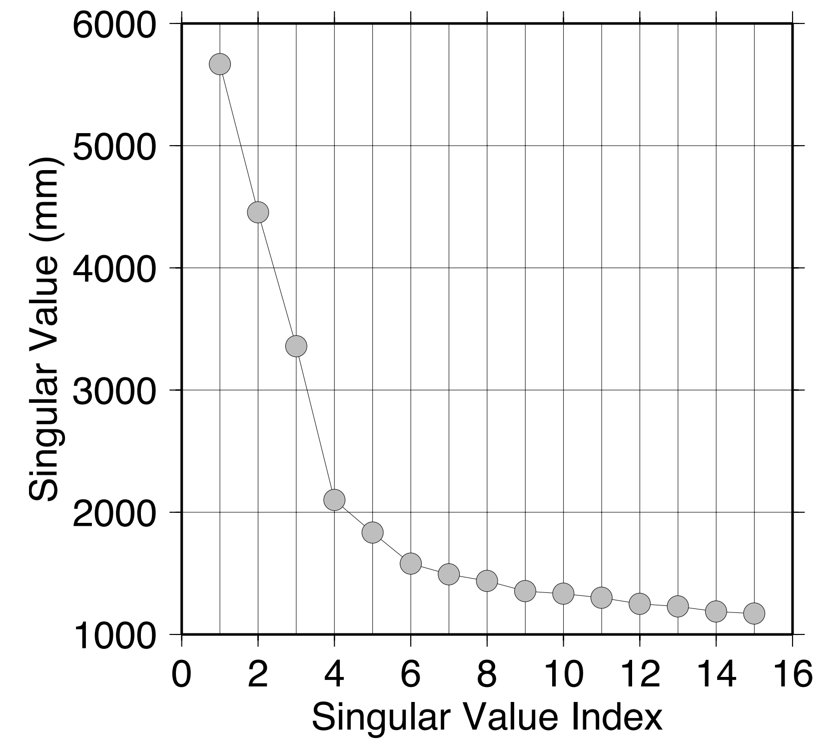

Singular Value (mm)
Singular Value Index
6000
5000
4000
3000
2000
1000
0
2
4
6
8
10
12
14
16

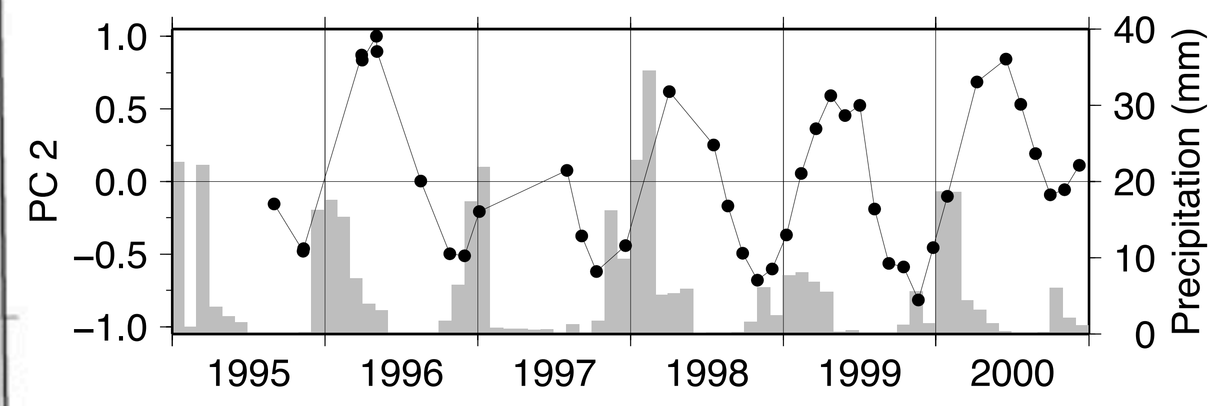

PC 1
Precipitation (mm)
1.0
0.5
0.0
−0.5
−1.0
40
30
20
10
0
1995
1996
1997
1998
1999
2000


38°
37.75°
37.5°
37.25°
237.5°
237.75°
238°
238.25°
mm
−10 −5 0 5 10 15 20
Displacement

PC 2

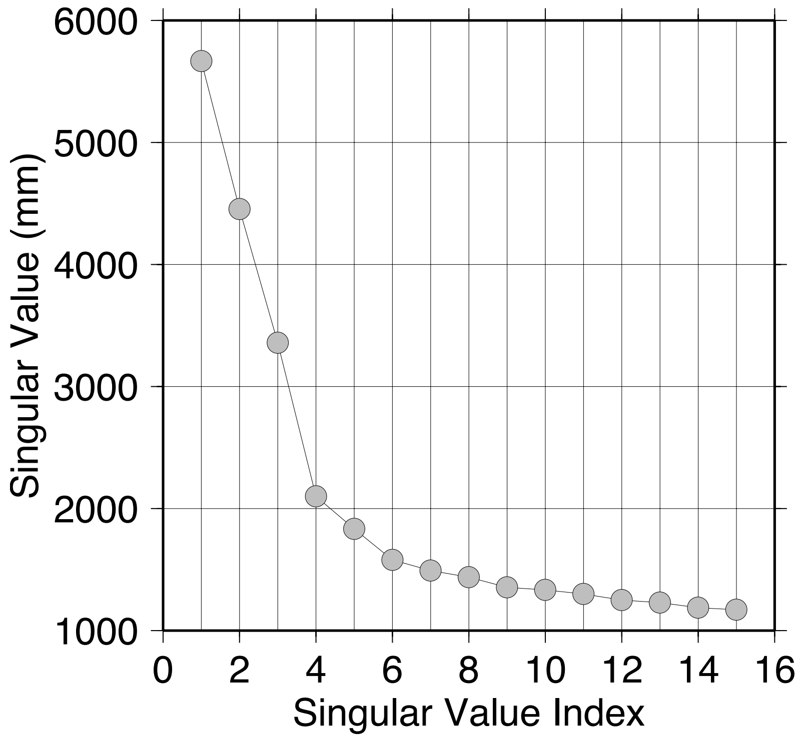


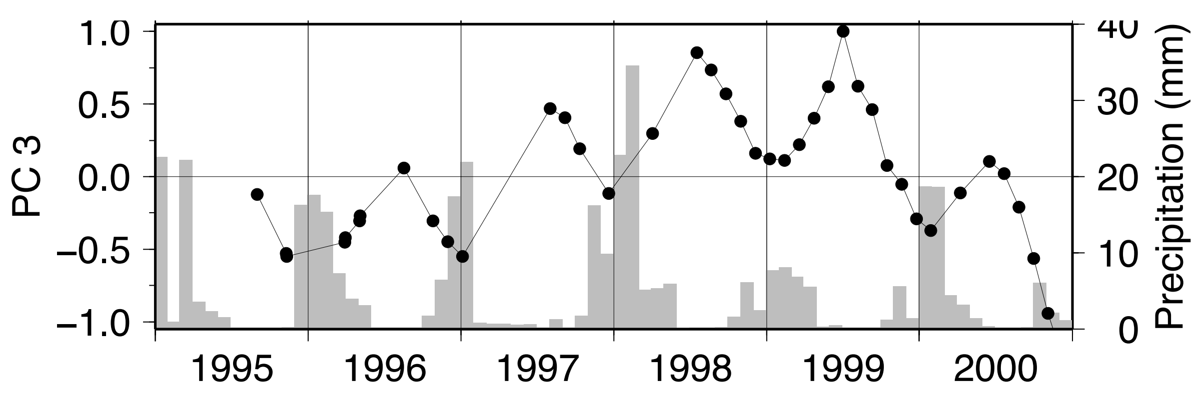


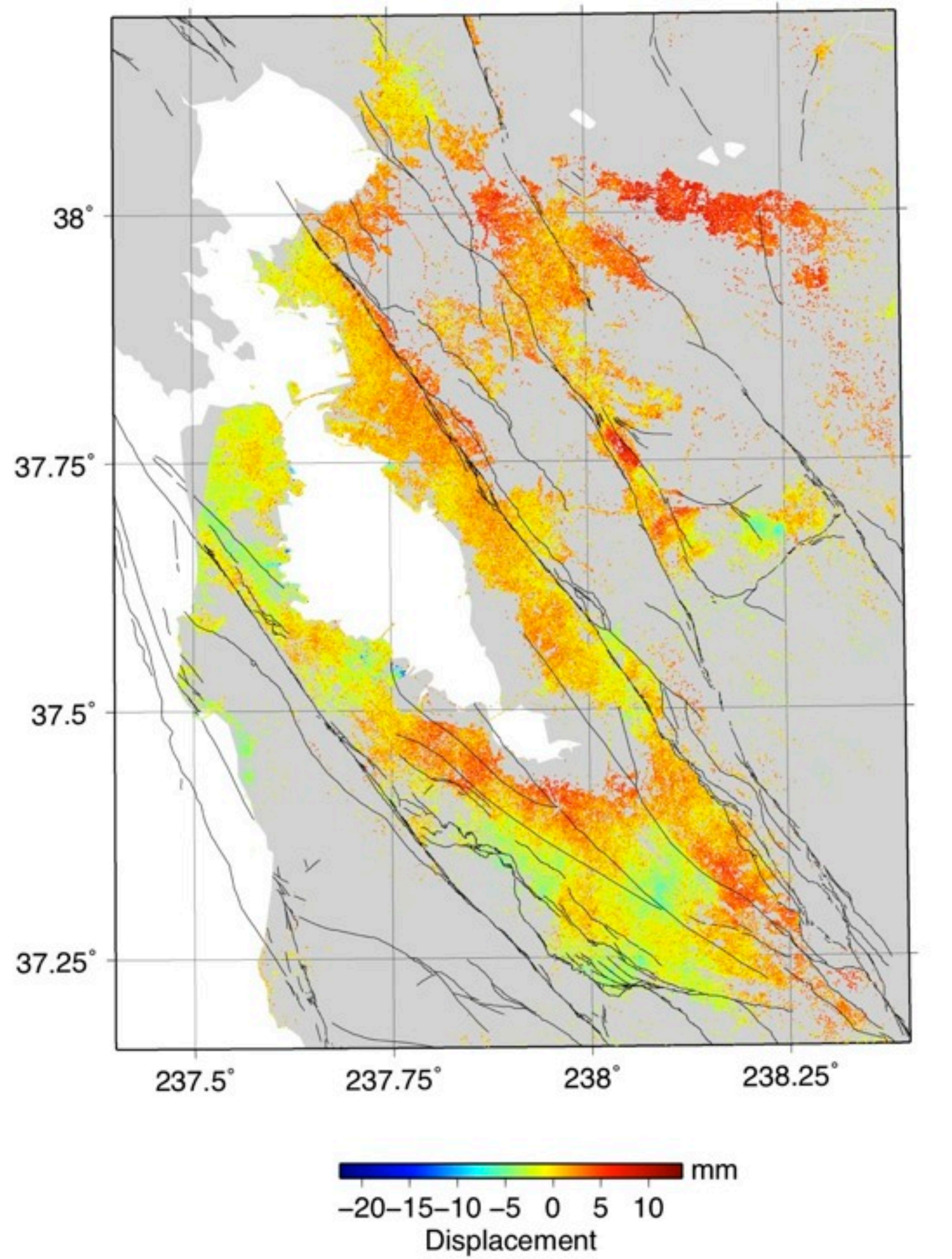


Lipovsky et al., *PCA and seasonal deformation*

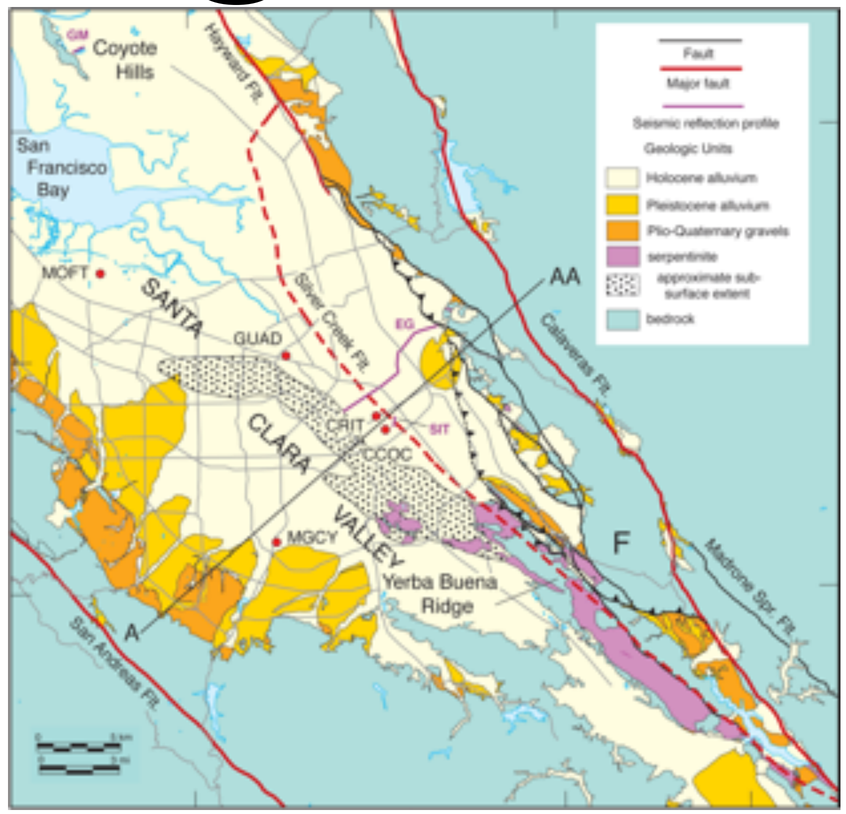
PC 3


Singular Value (mm)
Singular Value Index
6000
5000
4000
3000
2000
1000
0
2
4
6
8
10
12
14
16

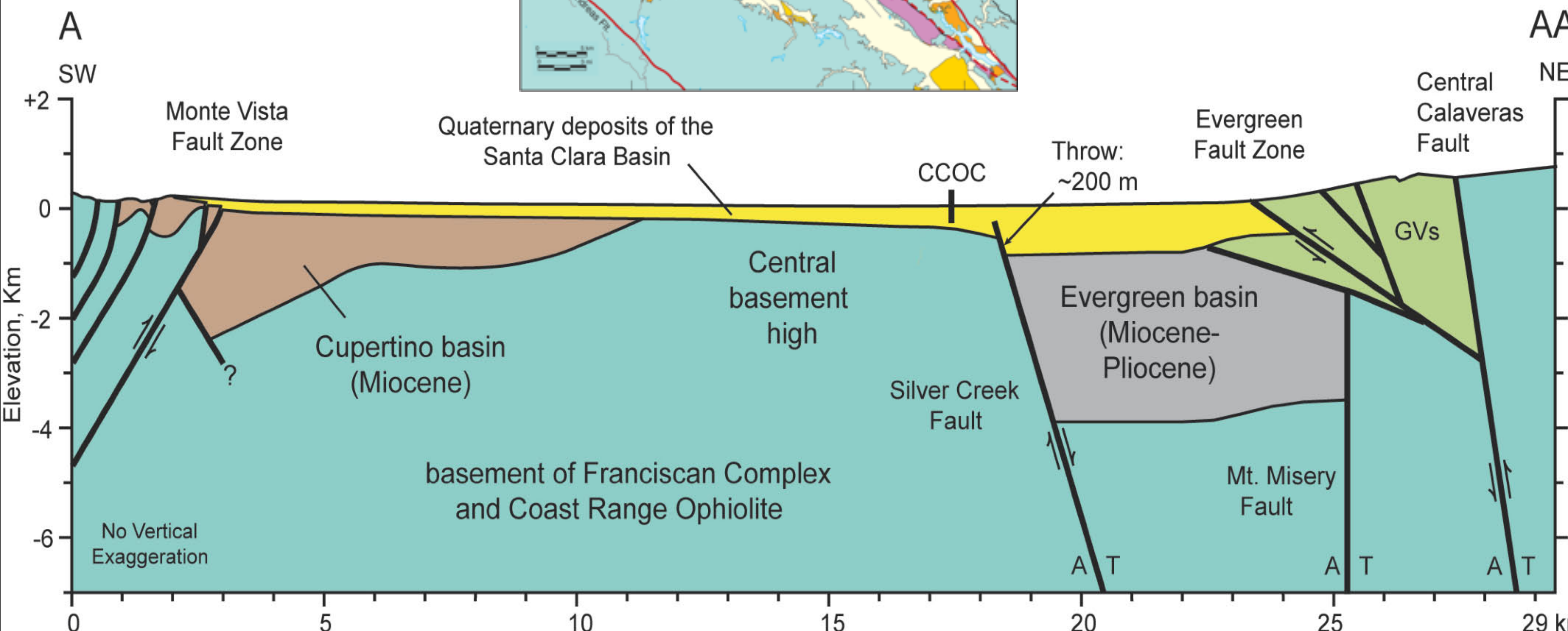

PC 3
Precipitation (mm)
1.0
0.5
0.0
−0.5
−1.0
40
30
20
10
0
1995
1996
1997
1998
1999
2000


38°
37.75°
37.5°
37.25°
237.5°
237.75°
238°
238.25°
mm
−20−15−10 −5 0 5 10
Displacement

Evergreen Basin

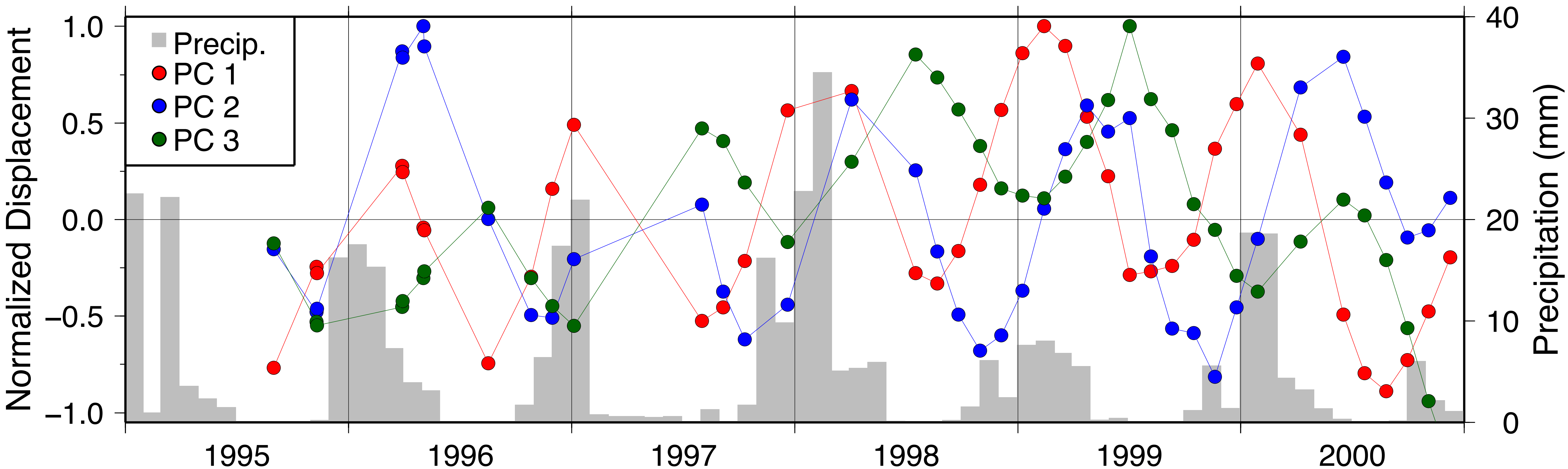


From Hansen et al 2010,
USGS OF1010



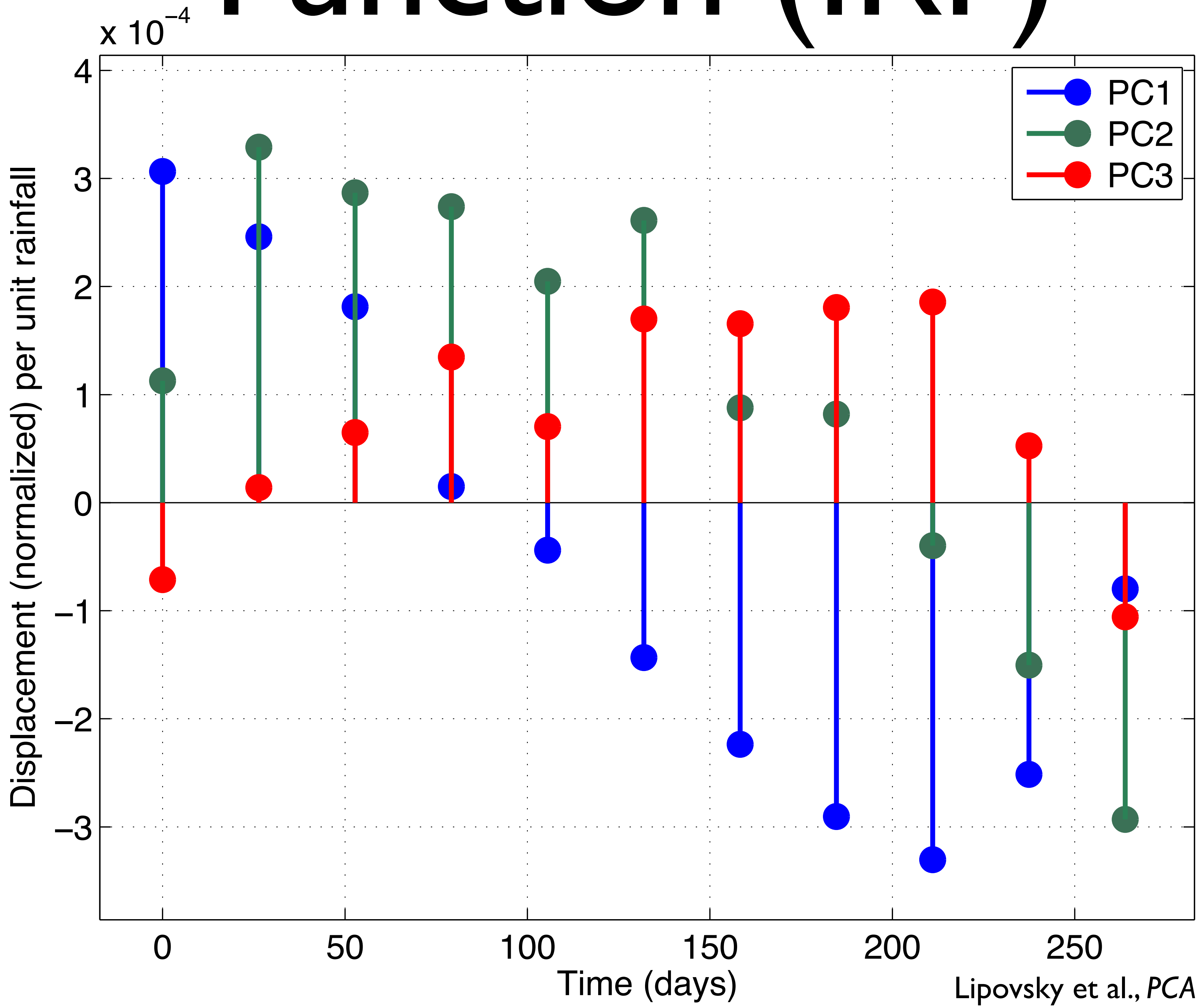
Lipovsky et al., *PCA and seasonal deformation*

Phase lags in PC time series



Lipovsky et al., *PCA and seasonal deformation*

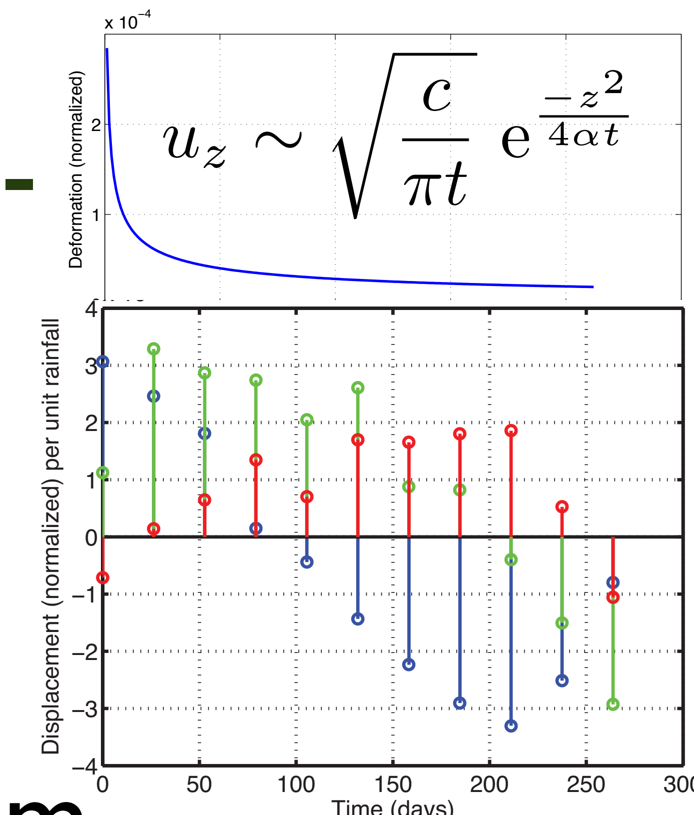
Data Impulse Response Function (IRF)



Lipovsky et al., *PCA and seasonal deformation*

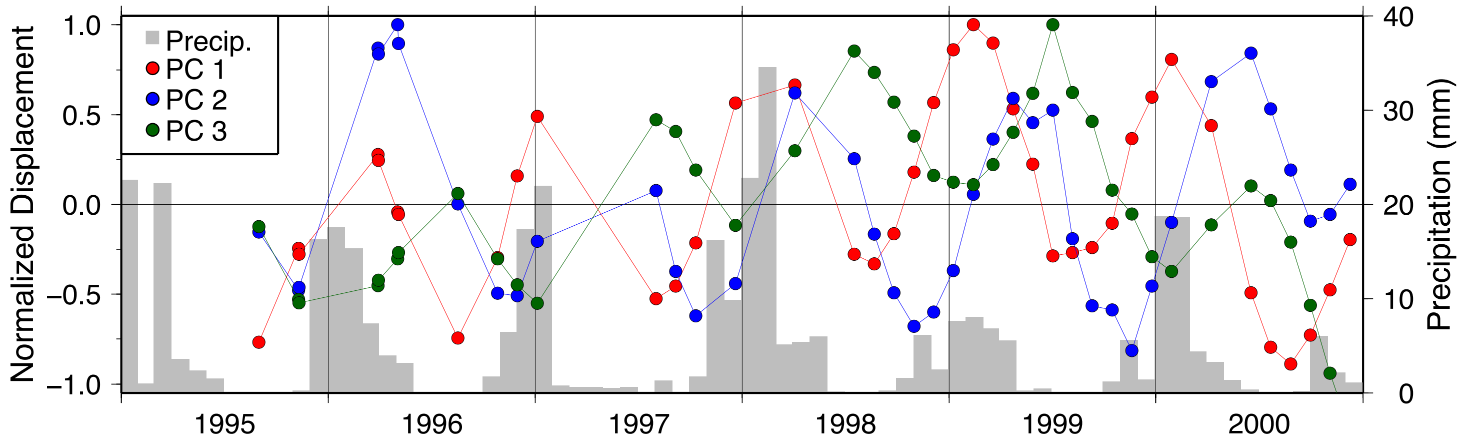
Data IRF as a "Residence Time Function"

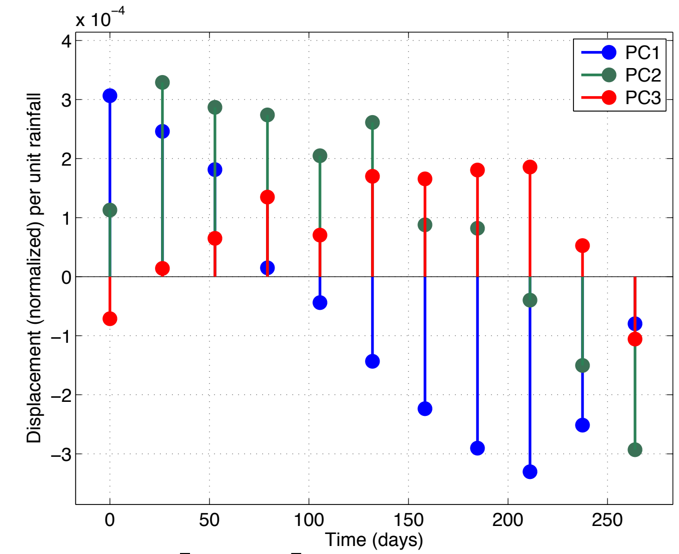
- The 1D poroelastic Greens function does not fit the observe precipitation-deformation relations.
- The bad fit arises from several not-applicable assumptions:
  - No lateral flow in any sense
  - Instant removal of water from system
- The *estimated* IRFs indicate how important these factors are, and show **how long precipitation remains in the system** causing deformation.

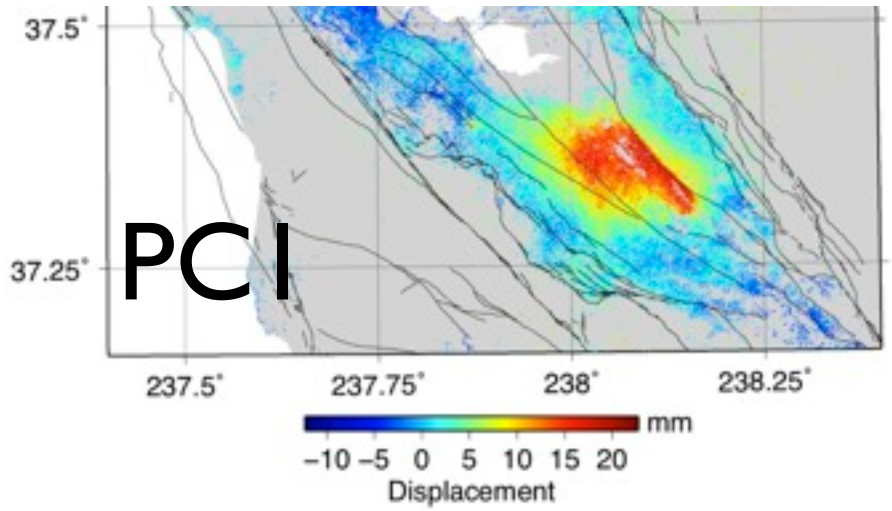


Lipovsky et al., *PCA and seasonal deformation*

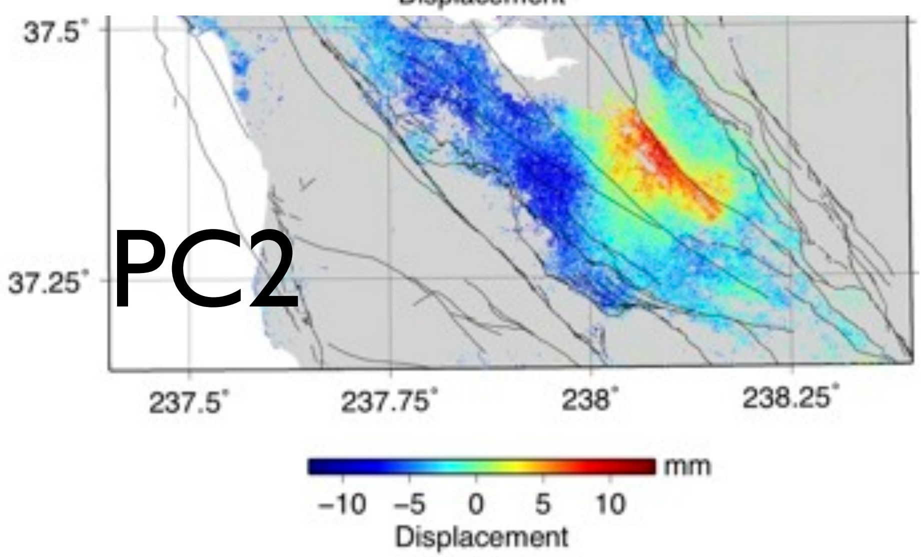
Application: shallow v. deep groundwater







The earliest seasonal phase is broad, whole-basin uplift.



The second seasonal phase occurs in the deeper part of the basin. *Mechanisms*: 1. longer residence time and/or 2. longer diffusion time

Lipovsky et al., *PCA and seasonal deformation*

Summary: An approach to modeling remote hydrology data

PCA is apt at detecting correlated patterns in large, complicated data sets. Additionally, under favorable circumstances, these patterns may be ascribed distinct physical interpretation. For the present data set this includes,

- Distinguishing between locations with shallow and deep groundwater.
- Identifying hydrologic deformations that occur with distinct annual phases.

Residence Time Functions show how long --and where-- water remains in a regional system after precipitation.

Future Work

1. Analytic expressions for surface displacements due to poroelastic diffusion in a "Long Graben" geometry, may be useful (Lipovsky et al, 2010b, in preparation).

2. Under certain circumstances, PCA can be used to distinguish between physical processes...

   Necessary and sufficient conditions for this association should prove highly valuable.

3. Test the "usefulness" hypothesis: talk to hydrologists

Questions?

http://waddle.ucr.edu/~brad/

1d Poroelastic Surface Loading Greens Function

$$u_z(z,t) = F\left(\frac{9(1-\nu_u)(\nu_u-\nu)}{2B^2(1+\nu_u)^2(1-\nu)}\right)\sqrt{\frac{\alpha}{\pi t}}\;\mathrm{e}^{\frac{-z^2}{4\alpha t}}$$

Impulse Estimation

Deformation

Unknown Coefficients

Precipitation

$$u(t_0) = \sum_{i=0}^{n} c_i \, r(t_i)$$

Possible mechanisms of deformation

Mechanism	Can explain seasonal phase?	Can explain spatial pattern?
• Earth tides	**Possibly**	**Not likely (?)**
• Poroelastic diffusion/expansion	**Possibly**	**Likely**
• Surface Loading	**Possibly**	**Not likely (?)**
• Seasonally correlated observational error	**Possibly**	**Possibly**

Poroelastic Plausibility

- α, Hydraulic Diffusivity = k/S_s (m^2/s)
  - 10^{-7} to 1 (derived from in situ measurements)
- Root-mean-square diffusive displacement

$$\sqrt{<\bar{x}^2> - <\bar{x}>^2} = \sqrt{2n\alpha t}$$

- After 25 days, RMS motion between 1m and 10^3 m

Frequency spectra of PC time series