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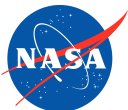
# Global Present-Day Surface Water Mass Transport and GIA from Joint Inversions of Multi- Satellite and Interdisciplinary Data

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With Contributions from

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A. Moore, and S. Owen***



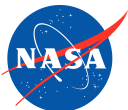
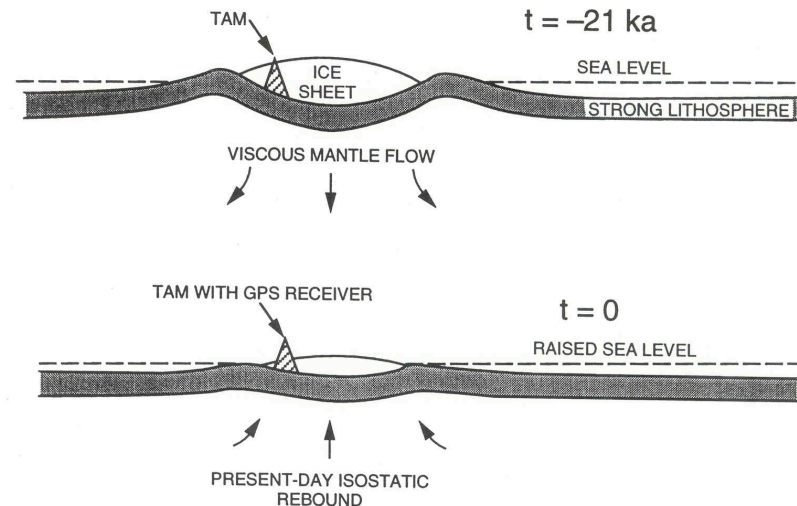
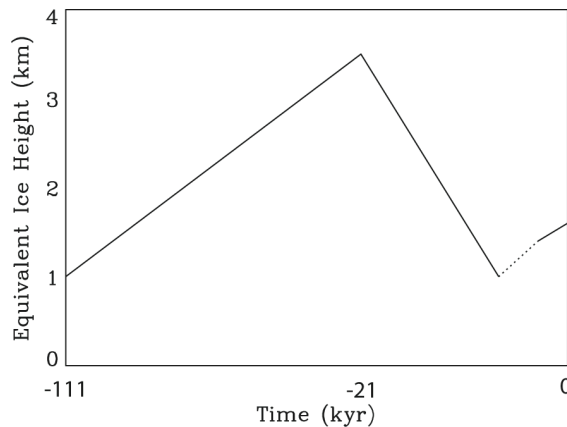


# Surface Mass Load and Solid Earth Deformation

## Two Time Scales:

$$\sigma(R_e, \vartheta, \varphi, t) = \sum_{n=1}^{\infty} \sum_{m=0}^n \sum_{q=c,s} M_{nmq}(t) Y_{nmq}(\vartheta, \varphi)$$

1. Short term - **Elastic** solid Earth - daily, seasonal, interannual  
Ocean tides, Hydrology, Ocean circulation
2. Secular trends - **Visco-elastic** solid Earth  $v\dot{E} = P + \tau\dot{P}$ 
  - Ice Mass Balance, Sea-Level Rise
  - Post-Glacial Rebound





# Periodic Surface Mass Change and Data

- Gravity
  - GRACE **No  $n=1$  terms**
  - SLR **very-low degrees**

$$V_{nmq} = \frac{4\pi a^2}{M_E} \frac{(1+k'_n)}{2n+1} M_{nmq}$$

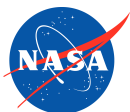
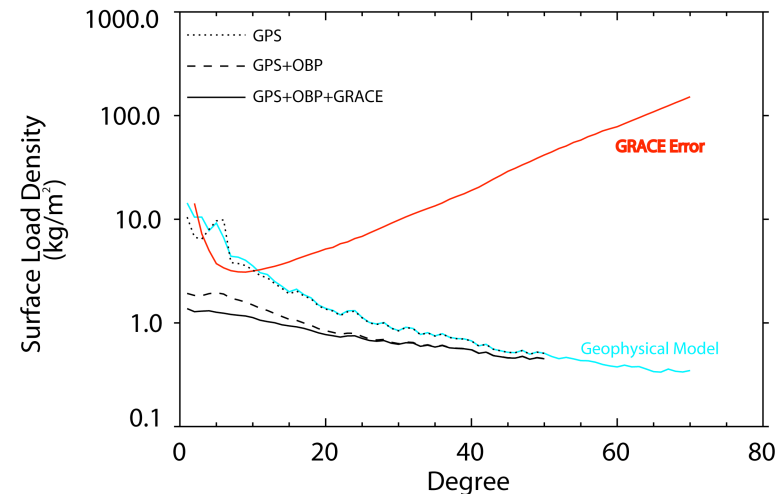
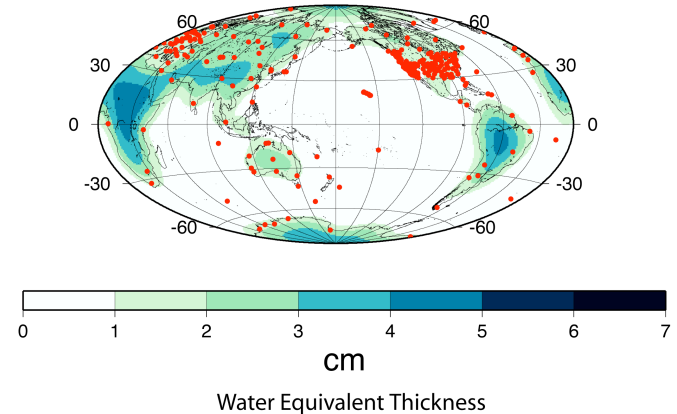
- Load Deformation (GPS, VLBI, SLR, InSAR)

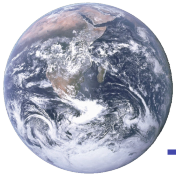
$$S(\vartheta, \varphi, t) = \frac{4\pi R_e^3}{M_E} \sum_{n=1}^{\infty} \sum_{m=0}^n \sum_{q=c,s} \frac{M_{nmq}(t)}{2n+1} [h'_n Y_{nmq} \hat{e}_r + l'_n \partial_{\vartheta} Y_{nmq} \hat{e}_{\vartheta} + l'_n \frac{1}{\sin \theta} \partial_{\varphi} Y_{nmq} \hat{e}_{\varphi}]$$

- Ocean Bottom Pressure, Topex/Jason assimilated

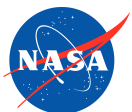
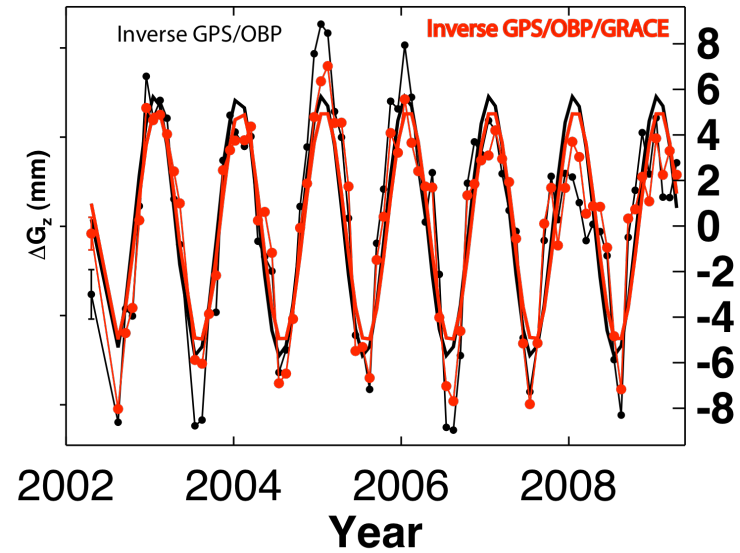
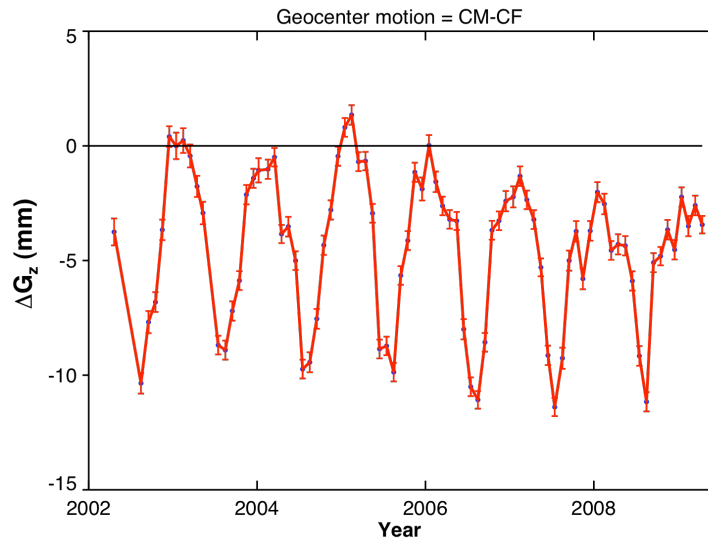
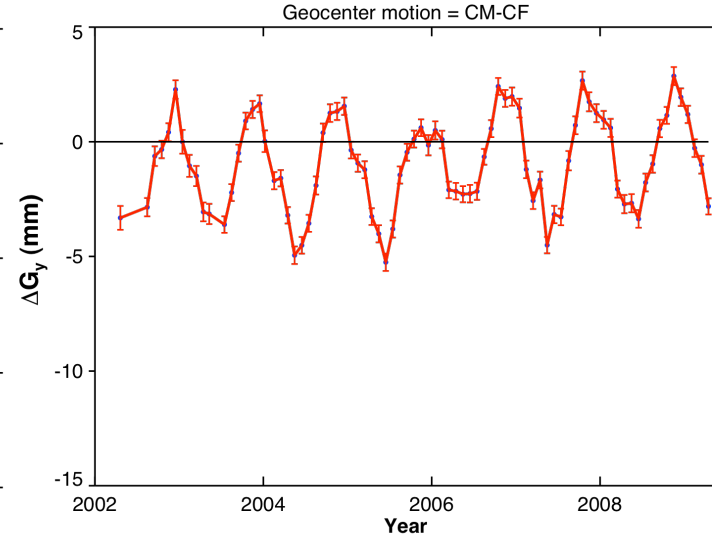
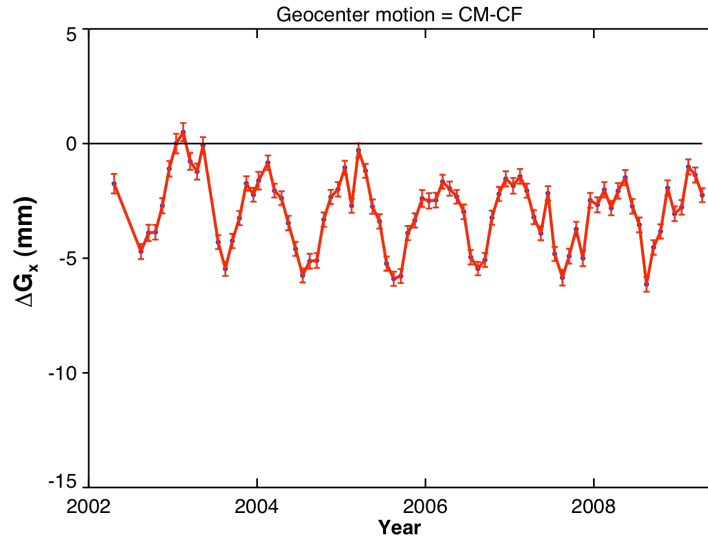
$$P(\vartheta, \varphi, t) = g \sum_{n=1}^{\infty} \sum_{m=0}^n \sum_{q=c,s} M_{nmq}(t) Y_{nmq} - P_{EQ}(M_{nmq})$$

Site Distribution and Monthly Uncertainty - GPS/OBP Inversion



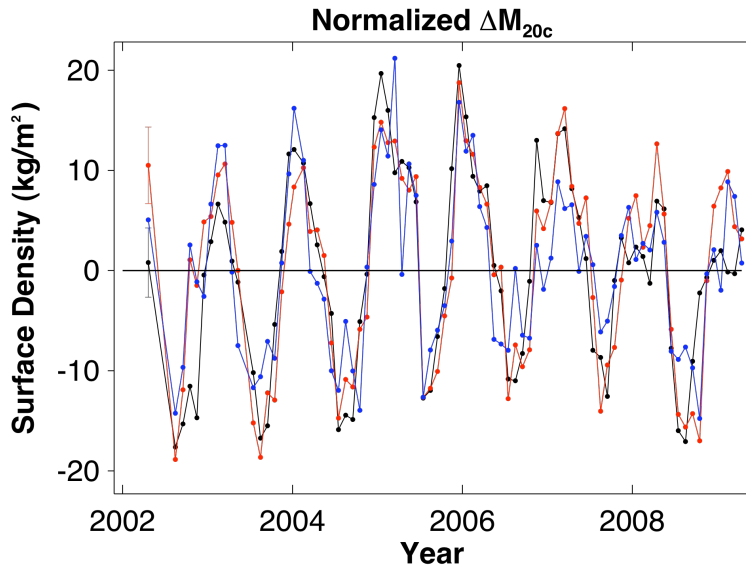


# Geocenter Motion





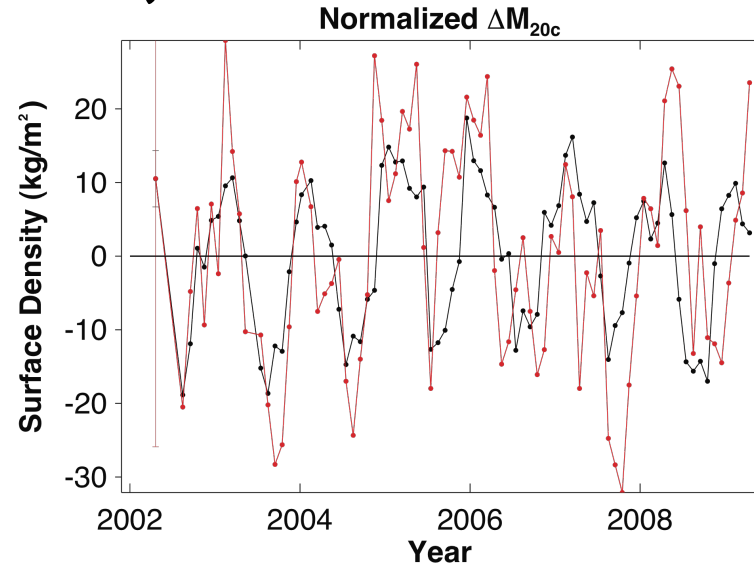
# Surface Density Coefficients from GPS/OBP, SLR, GRACE



GPS/OBP

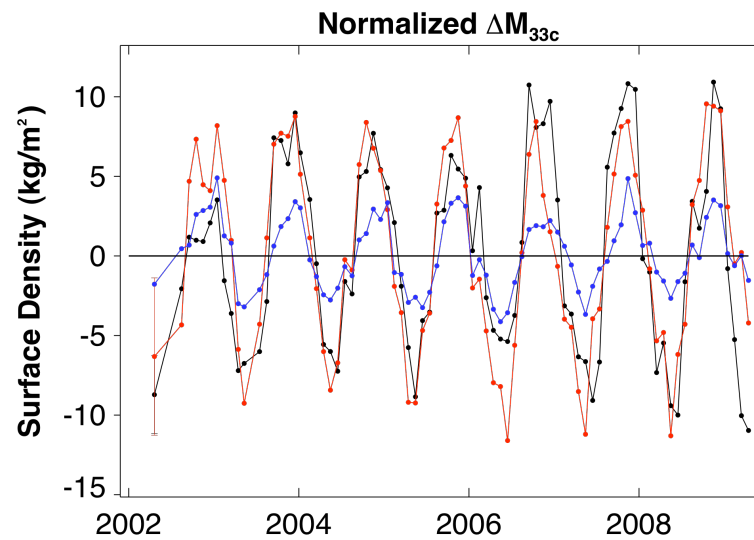
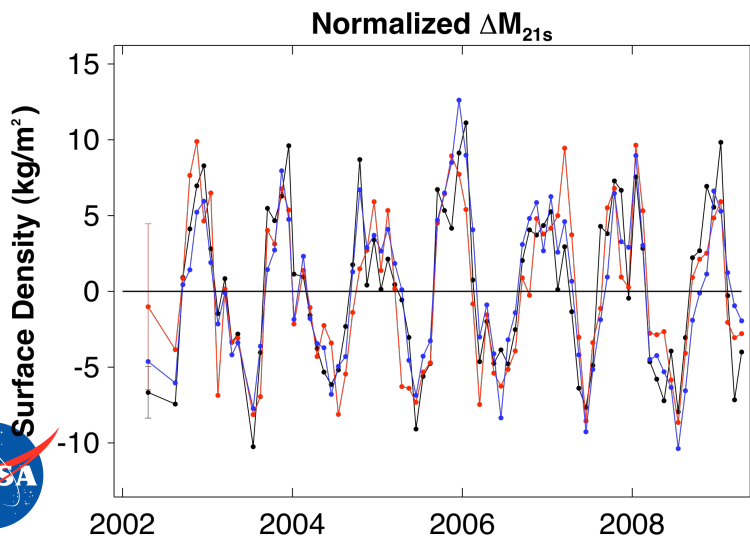
JPL GRACE

SLR



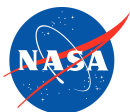
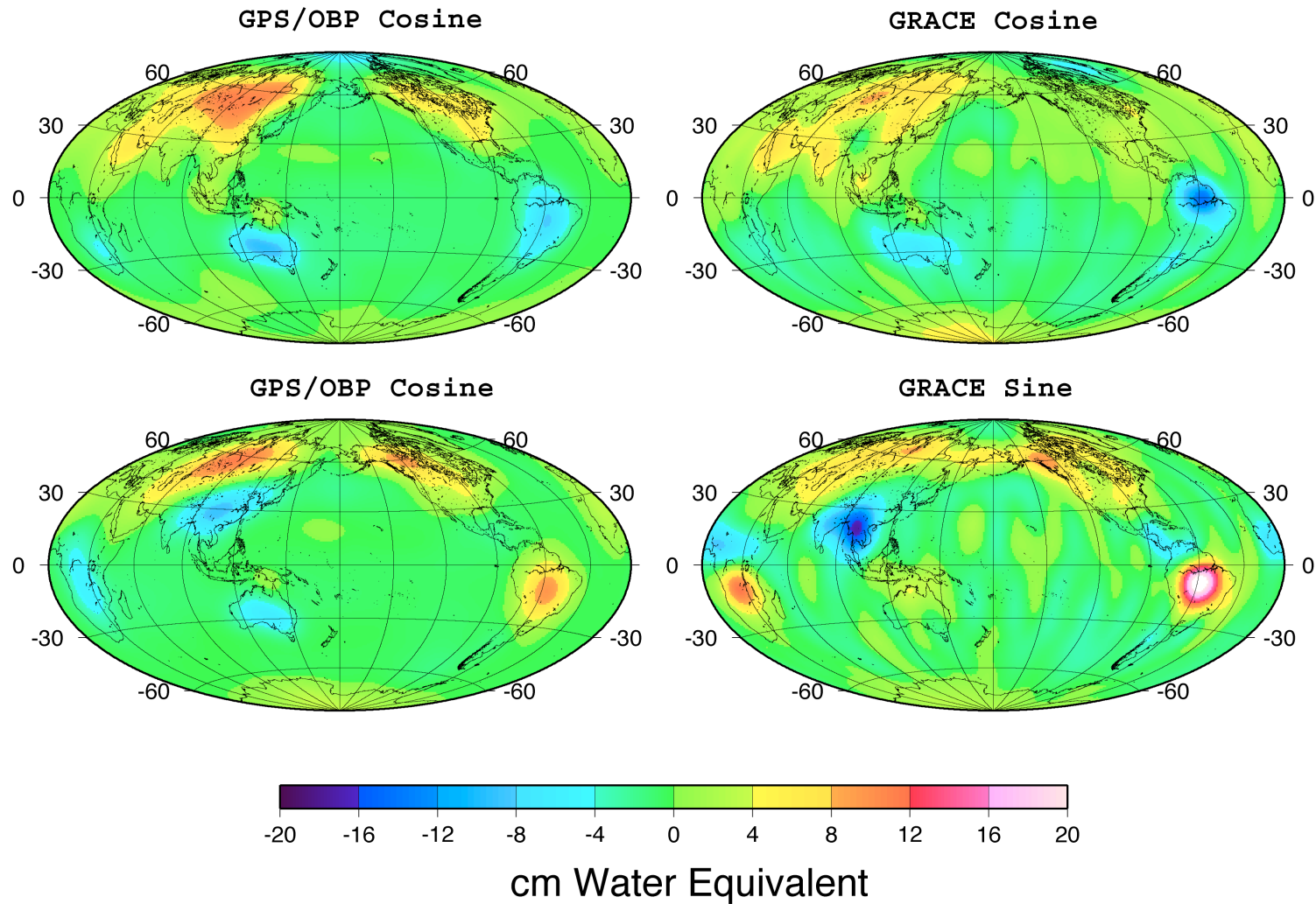
SLR

CSR GRACE





# Compare GPS/OBP and GRACE Annual Variations (2004-2005)



351 Day GPS Draconitic Harmonic error ?



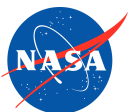


# Secular Inversion – Separating Present-day Surface Mass Trend and GIA

- Geodetic Data are sensitive to both Present-Day Surface Water Mass Trend (PDMT) and GIA (PGR)

$$\begin{aligned} \dot{\vec{s}}_i - \dot{\vec{s}}_{\text{cm}} = & \sum_{n=1}^{\infty} \sum_{mq} \left( (a_n \dot{M}_{nmq} + b_n \dot{M}_{nmq}^{v,h}) Y_{nmq} \hat{\mathbf{e}}_r + (c_n \dot{M}_{nmq} + d_n \dot{M}_{nmq}^{v,l}) \left( \partial_{\vartheta} Y_{nmq} \hat{\mathbf{e}}_{\vartheta} + l'_n \frac{1}{\sin \vartheta} \partial_{\varphi} Y_{nmq} \hat{\mathbf{e}}_{\varphi} \right) \right) \\ & - \frac{4\pi}{\sqrt{3}} \frac{a^3}{M_E} \left( \dot{M}_{11c} \hat{\mathbf{e}}_x + \dot{M}_{11s} \hat{\mathbf{e}}_y + \dot{M}_{10c} \hat{\mathbf{e}}_z \right) + \vec{\omega}_p \times \vec{\mathbf{X}}_i \end{aligned}$$

- Both GIA and PDMT models contain unknown and potentially large errors
- Multiple data sets (GRACE, GPS and ECCO Ocean Model) needed to separate PDMT and GIA
- Secular geocenter motion due to PDMT and GIA have not been reliably determined from observations.





## Previous Methods of Separation

- Kinematic/Iterative:**

*Wahr et al., 2000*

$$\dot{M}_{nmq}^{v,k} \approx \frac{2}{2n+1} \dot{M}_{nmq}^{v,h}$$

$$\text{Vertical: } \dot{U} = \sum_{n=1}^{\infty} \sum_{mq} (a_n \dot{M}_{nmq} + b_n \dot{M}_{nmq}^{v,h}) Y_{nmq}$$

$$\text{Geoid: } \dot{N} = \sum_{n=2}^{\infty} \sum_{mq} (f_n \dot{M}_{nmq} + b_n \dot{M}_{nmq}^{v,k}) Y_{nmq}$$

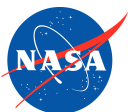
- **Not very accurate** for  $n = 2-7$ ; Exploits only 1 physical property of the viscoelastic Earth but not other a priori information
- **Not applicable** for  $n=1$ , **horizontal motion**

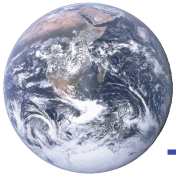
- Dynamic:**

*Wu et al., 2002*

$$\mathbf{D} = \mathbf{F}(\dot{\mathbf{M}}, \mathbf{M}_{\text{past}}, \mathbf{H}_L, \mathbf{v}, \dots)$$

- **Non-linear** Inverse problem
- Modern geodesy **not enough to resolve ice history**





# Kinematic Global Inverse Methodology

$$\mathbf{Y} = \mathbf{A}\dot{\mathbf{M}}_{nmq} + \mathbf{B}\dot{\mathbf{M}}_{nmq}^{v,l} + \mathbf{C}\dot{\mathbf{M}}_{nmq}^{v,h} + \mathbf{D}\dot{\mathbf{M}}_{nmq}^{v,k} + \mathbf{E}\omega$$

$$\dot{M}_{nmq}^{v,k} \approx \frac{2}{2n+1} \dot{M}_{nmq}^{v,h}$$

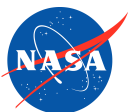
Dynamically consistent and conservative a priori info



$$\dot{M}_{nmq}^{v,h} = \sum_j (-s_{nj}) r_{nj}^h \int_{-\infty}^t M_{nmq}(\tau) e^{s_{nj}(\tau-t)} d\tau, \quad \dot{M}_{nmq}^{v,l} = \sum_j (-s_{nj}) r_{nj}^l \int_{-\infty}^t M_{nmq}(\tau) e^{s_{nj}(\tau-t)} d\tau .$$

- **Solution Strategy:**

- Least squares under SVD with reduced a priori
- Dynamically constructed a priori GIA model with large uncertainties
- Full data and a priori covariance matrices
- No a priori information used for PDMT

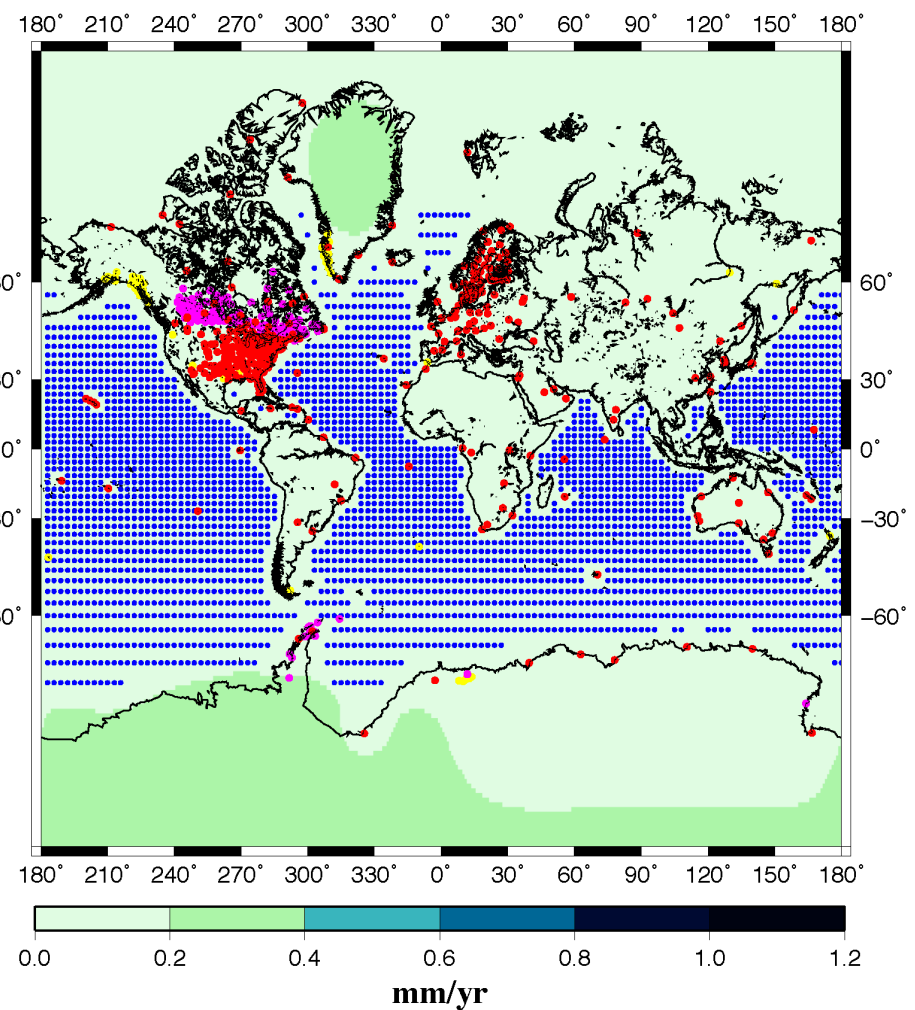
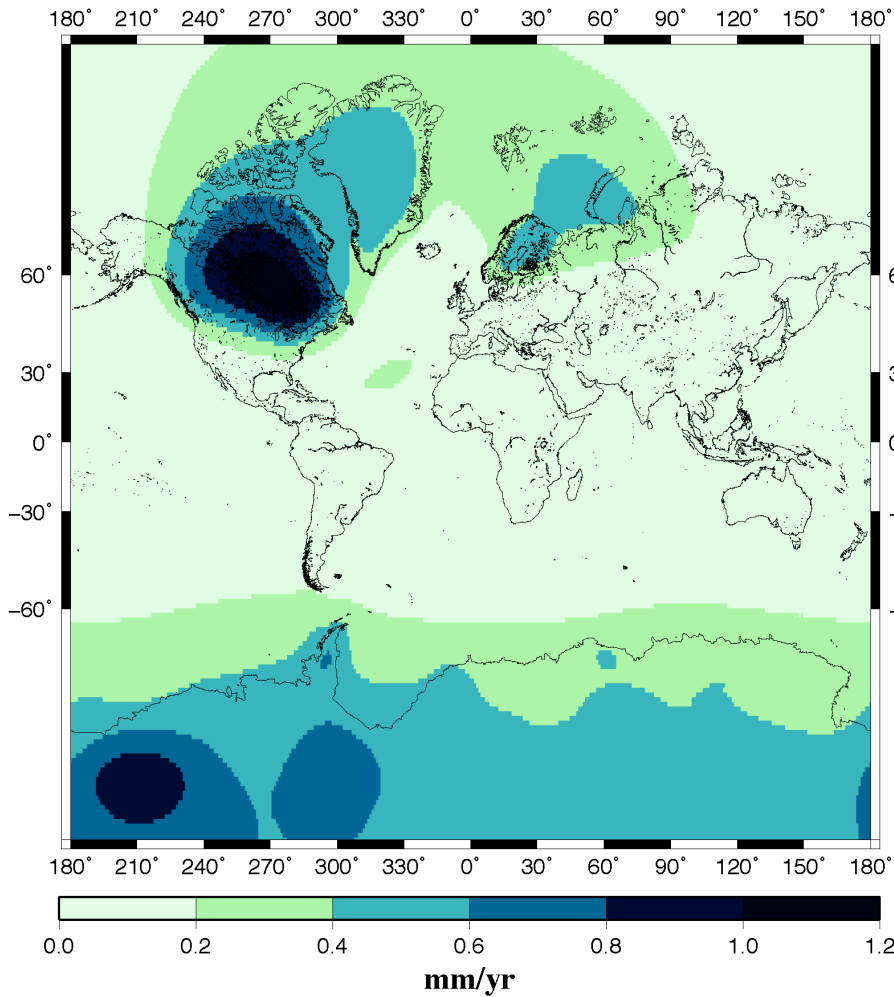


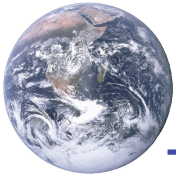


# Velocity and OBP Data and GIA Geoid Uncertainty

## A Priori

## Posteriori





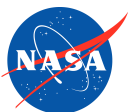
## Results: PDMT and GIA

Theoretical Secular Geocenter Motion:

$$\begin{aligned} \dot{\mathbf{S}}_{cm} - \dot{\mathbf{S}}_{cf} = & \frac{4\pi}{\sqrt{3}} \frac{R_e^3}{M_e} \left[ \left( 1 - \frac{h'_1 + 2l'}{3} \right) (\dot{M}_{11c} \hat{e}_x + \dot{M}_{11s} \hat{e}_y + \dot{M}_{10c} \hat{e}_z) \right] \\ & - \frac{4\pi}{\sqrt{3}} \frac{R_e^3}{M_e} \left[ \frac{\dot{M}_{11c}^{v,h} + 2\dot{M}_{11c}^{v,l}}{3} \hat{e}_x + \frac{\dot{M}_{11s}^{v,h} + 2\dot{M}_{11s}^{v,l}}{3} \hat{e}_y + \frac{\dot{M}_{10c}^{v,h} + 2\dot{M}_{10c}^{v,l}}{3} \hat{e}_z \right] \end{aligned}$$

Table 1 Long-Wavelength Signatures

| Component                   | PDMT Estimate                   | GIA Estimate                     |
|-----------------------------|---------------------------------|----------------------------------|
| $\mathbf{V}_{Gx}$ (mm/yr)   | $-0.08 \pm 0.04$                | $-0.10 \pm 0.01$                 |
| $\mathbf{V}_{Gy}$ (mm/yr)   | $0.29 \pm 0.05$                 | $0.11 \pm 0.02$                  |
| $\mathbf{V}_{Gz}$ (mm/yr)   | $-0.16 \pm 0.07$                | $-0.72 \pm 0.06$                 |
| $\dot{\mathbf{j}}_2$ (1/yr) | $(5.2 \pm 0.5) \times 10^{-11}$ | $(-3.4 \pm 0.4) \times 10^{-11}$ |
| $\dot{\mathbf{j}}_3$ (1/yr) | $(1.3 \pm 0.4) \times 10^{-11}$ | $(0.6 \pm 0.3) \times 10^{-11}$  |





# Point and Regional Averages

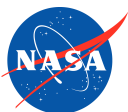
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$$\bar{G}(\Omega_0) = \sum a_{nmq} G_{nmq} = \iint A(\Omega, \Omega_0) G(\Omega) d\Omega + \sum a_{nmq} \Delta_{nmq}$$

- Optimal Kernel using Geophysical inverse
- Optimal with a priori: find  $a_{nmq}$  that minimizes MSE

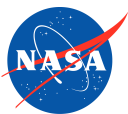
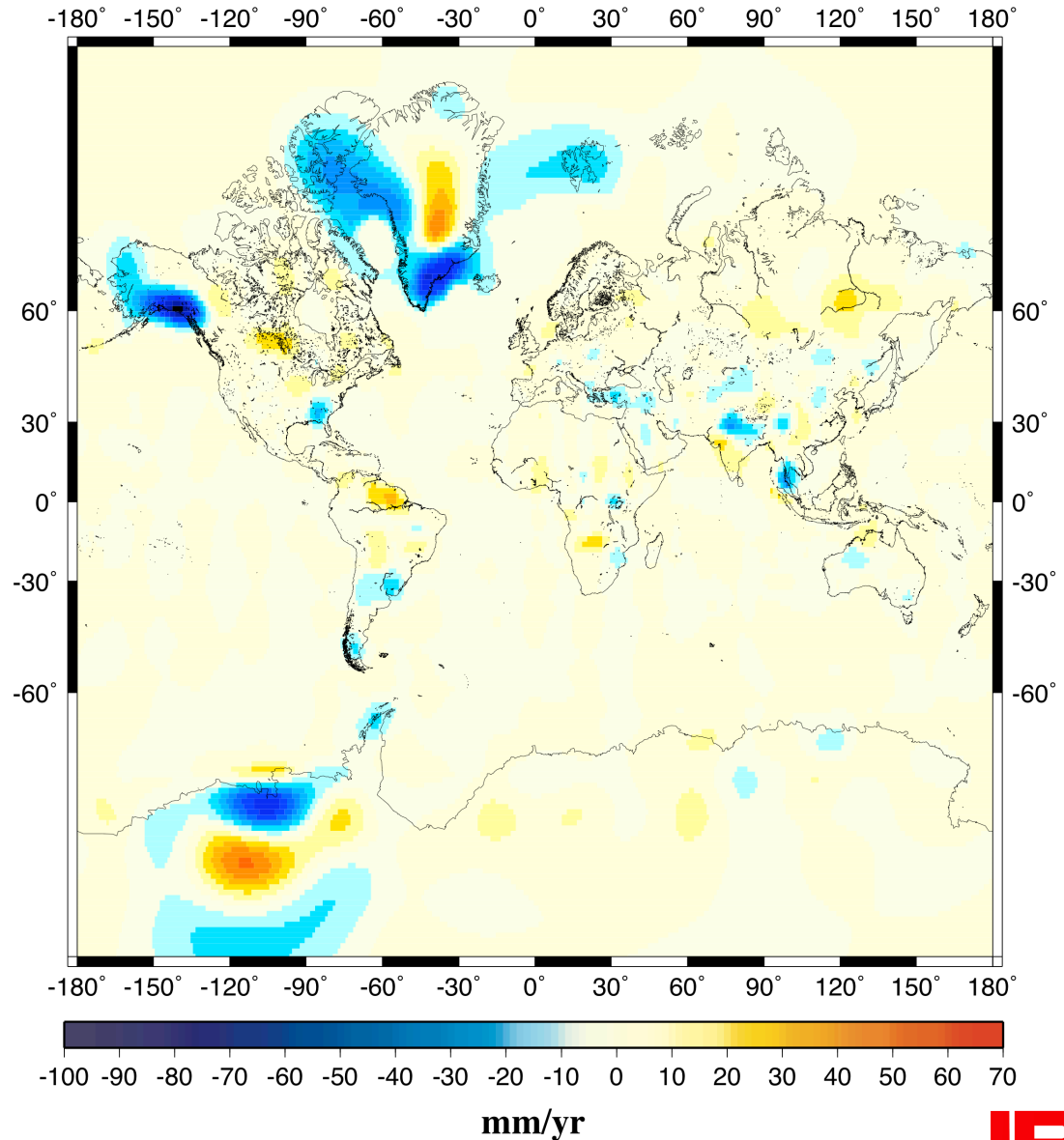
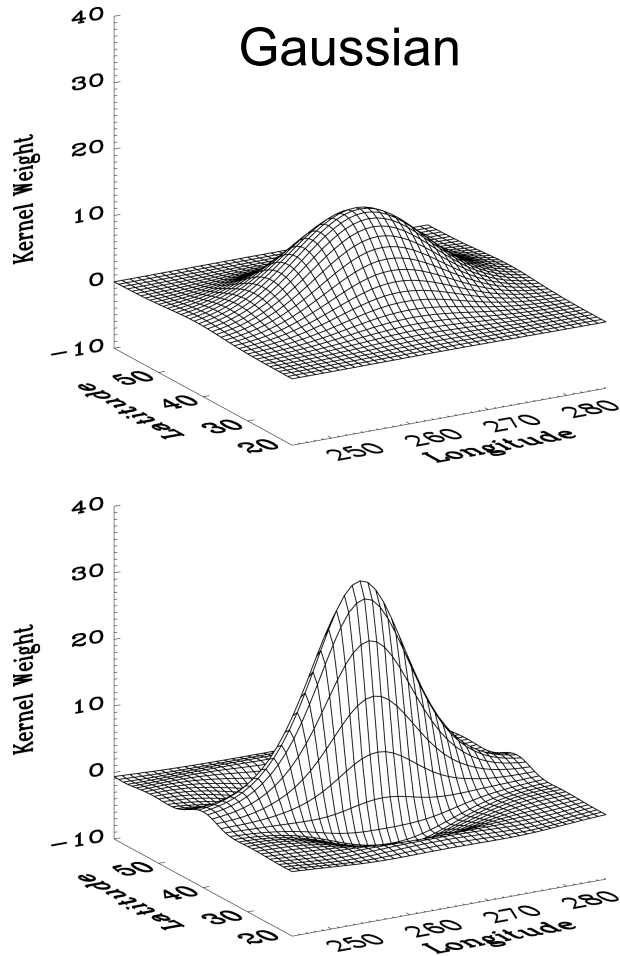
$$\hat{X} = X_0 + \Sigma_G (\Sigma_G + \Sigma)^{-1} (X - X_0)$$

- No Kaula's rule for time-variable gravity
- Isotropical Gaussian covariance function
- How to quantify a priori information is very tricky and dangerous



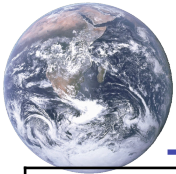


# Results – PDMT in Water Equivalent Thickness (Atmosphere Included)



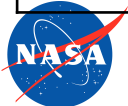
Anisotropic Optimal Kernel





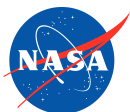
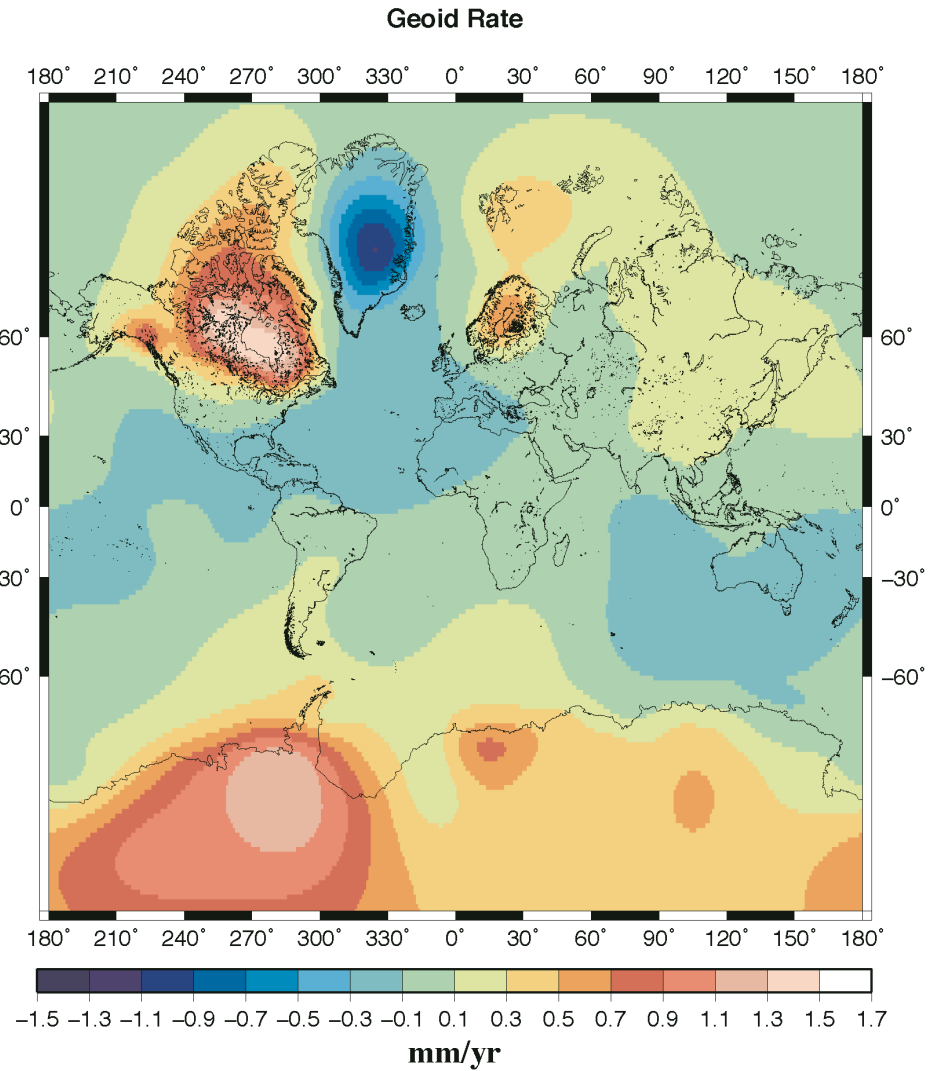
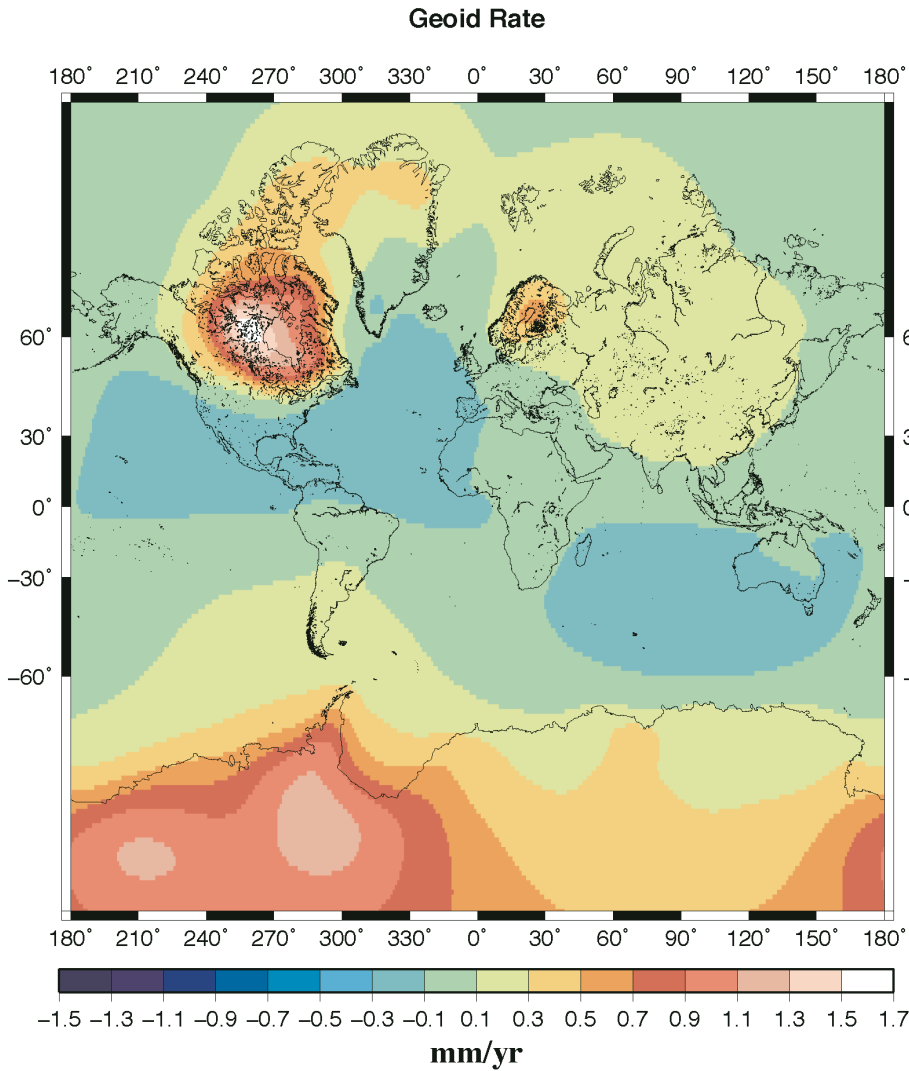
## Results: PDMT (Atmosphere Included)

| Region                       | Mass Gain (GT/y)<br>(GIA corrected) | Mass Gain (GT/y)<br>(GIA estimated) | Atmospheric<br>Mass Gain (Gt/y) |
|------------------------------|-------------------------------------|-------------------------------------|---------------------------------|
| Greenland                    | - 161 ± 35                          | - 104 ± 23                          | 4                               |
| Alaska/Yukon                 | - 68 ± 28                           | - 101 ± 23                          | -6                              |
| West Antarctica              | - 99 ± 59                           | - 64 ± 32                           | -5                              |
| East Antarctica              | 13 ± 54                             | - 23 ± 29                           | -26                             |
| Land                         | - 36 ± 148                          | 14 ± 76                             | -106                            |
| The Arctic                   | - 69 ± 24                           | - 68 ± 16                           | 1                               |
| Ocean                        | 341 ± 137                           | 236 ± 81                            | 32                              |
| Non-steric<br>mean sea level | 0.82 ± 0.4 mm/y                     | 0.54 ± 0.3 mm/y                     | Global oceans                   |
| Non-steric<br>mean sea level | 0.94 ± 0.4 mm/y                     | 0.70 ± 0.3 mm/y                     | ±66°                            |





# A Priori and Posteriori GIA Geoid Trend



ICE-5G/IJ2005/VM2

Inverted





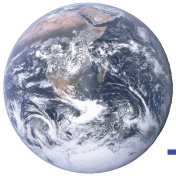
# Signature Separation and Perspectives

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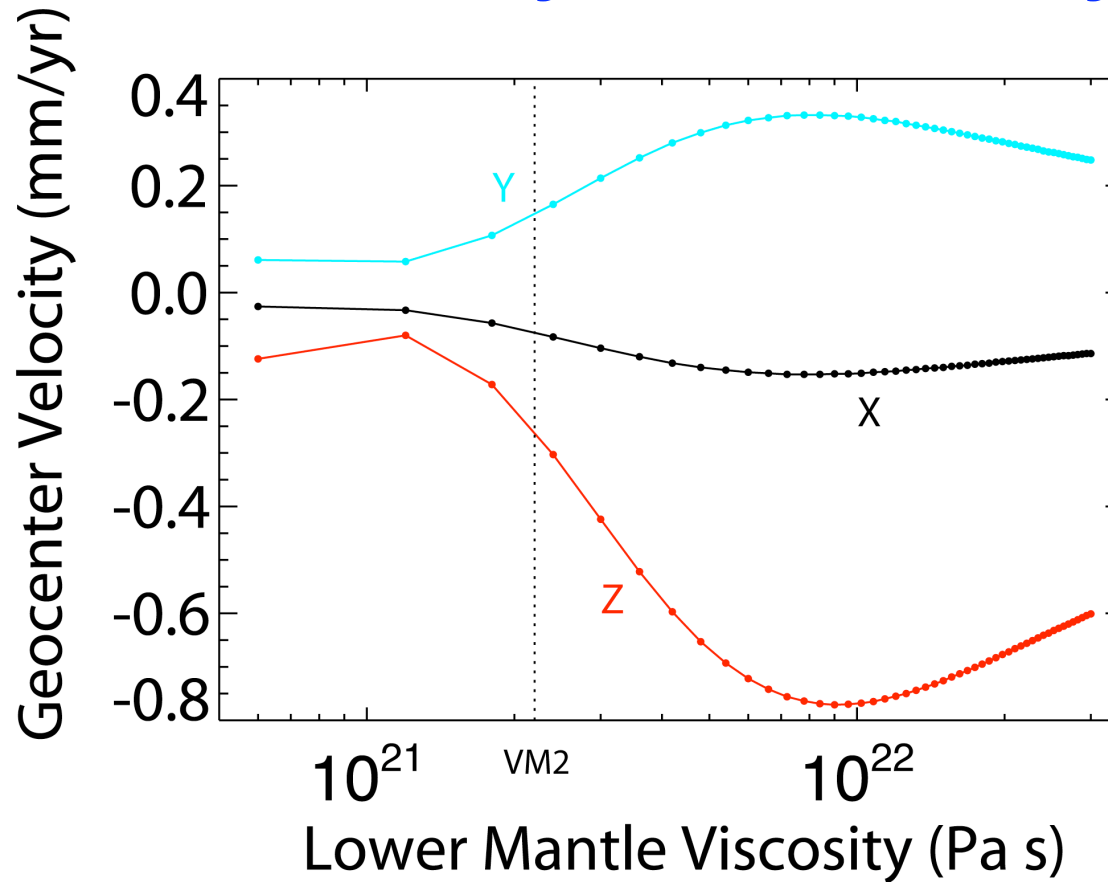
- **Global Kinematic:**
  - Use **fixed** deglaciation **areas** and intrinsic **dynamics**
  - Successfully **separates long-wavelength** signatures
  - **Fail** to incorporate historical relative sea level data
  - Lack dual data coverage over ice sheet interiors
  - Cannot resolve ice history and Earth rheology
  - Full dynamics requires sea-level eqn solver with rev. rotational feedback
- **Dynamic:** Massive combination of all data

$$D = F(\dot{M}, M_{\text{past}}, H_L, v, \dots)$$





# GIA Geocenter Motion – Lower Mantle Viscosity and Ice History



- Geocenter velocity also depends on Ice history

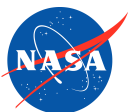




## Summary

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- Separate Estimates of PDMT and GIA Coefficients and Cov
- Secular geocenter motion due to PDMT and GIA determined to  $< \pm 0.1$  mm/yr
- Significant GIA geoid trends for Greenland (-) and Alaska (+)
- Geodetic coverage in Greenland and Antarctica is still poor
  - Add ice altimetric data + More accurate rotational feedback
  - Dynamic inversion with relative sea level data
- Although challenging, GRACE has the potential to determine non-steric mean sea-level change to  $\pm 0.1$  mm/a
  - Reduce atmospheric model error
  - Better low-degree trends: Joint processing of SLR and GRACE data

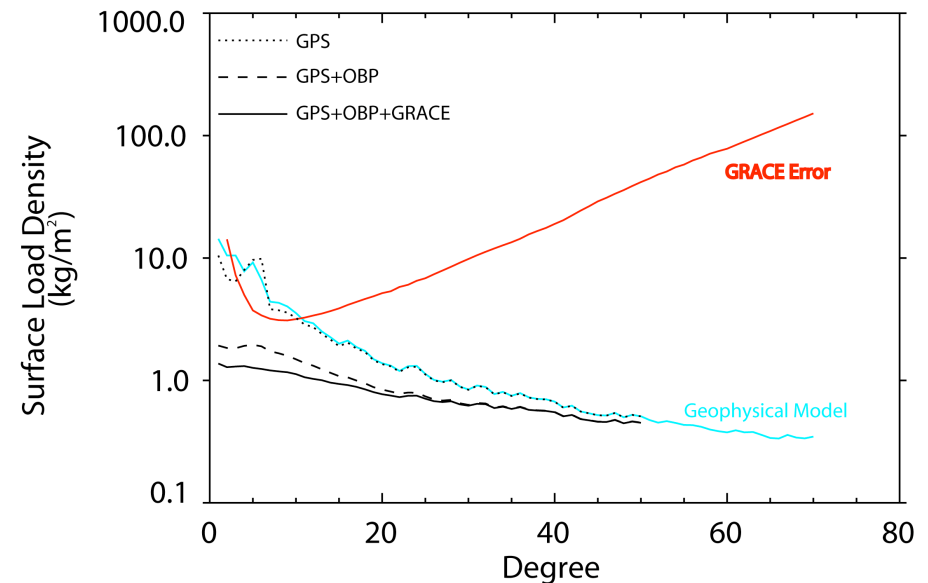




# Global SVD/LSE/BLE Inverse Methodology

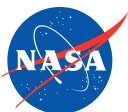
$$G(\vartheta, \varphi, t) = \sum_{nmq} G_{nmq}(\vartheta, \varphi, p..) M_{nmq}(t)$$

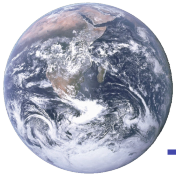
- **Parameters in spherical harmonic domain**
- **SVD decomposition**
- **LSE of Good data + Filtering bad data with reduced a priori**
- **High degree and order**
- **Robust low degree results**
- **Noisy individual high degree coefficients**
- **Accurate linear combinations for regional recovery**



## Issues :

- Direct Geocenter info, and Scale
- A priori (regularization) or not and starting degree
- Systematic Errors





# Measurement Equations

Geocentric velocity at 664 SLR/VLBI/GPS sites :

$$\dot{\mathbf{s}}_i - \dot{\mathbf{s}}_{\text{cm}} = \sum_{n=1}^{\infty} \sum_{mq} \left( (a_n \dot{M}_{nmq} + b_n \dot{M}_{nmq}^{v,h}) Y_{nmq} \hat{\mathbf{e}}_r + (c_n \dot{M}_{nmq} + d_n \dot{M}_{nmq}^{v,l}) (\partial_{\vartheta} Y_{nmq} \hat{\mathbf{e}}_{\vartheta} + l'_n \frac{1}{\sin \vartheta} \partial_{\varphi} Y_{nmq} \hat{\mathbf{e}}_{\varphi}) \right) - \frac{4\pi}{\sqrt{3}} \frac{a^3}{M_E} (\dot{M}_{11c} \hat{\mathbf{e}}_x + \dot{M}_{11s} \hat{\mathbf{e}}_y + \dot{M}_{10c} \hat{\mathbf{e}}_z) + \vec{\omega}_p \times \vec{\mathbf{X}}_i$$

$$\dot{\mathbf{s}}_i - \dot{\mathbf{s}}_{\text{mean}} = \sum_{mq} \alpha_{1mq} (\dot{M}_{1mq}^{v,h} - \dot{M}_{1mq}^{v,l}) + \sum_{n=1} \sum_{mq} \dots\dots\dots$$

GRACE Geoid up to degree 60 :

$$\dot{N} = \sum_{n=2}^{\infty} \sum_{mq} (f_n \dot{M}_{nmq} + b_n \dot{M}_{nmq}^{v,k}) Y_{nmq}$$

JPL ECCO Ocean Bottom Pressure  $3^{\circ} \times 3^{\circ}$  :

$$\dot{P} = \sum_{n=1} \sum_{mq} g O(\vartheta, \varphi) \dot{M}_{nmq} Y_{nmq} - g \dot{M}_{EQ}, \quad \dot{M}_{EQ} = G(\dot{M}_{nmq}, O_{nmq})$$

