Global Inverse for Surface Mass Variations, Geocenter Motion, and Earth Rheology

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Objectives

• Seasonal to Interannual Mass Variation: Elastic Earth
  - Multi-satellite data combination to achieve complete global coverage
    GRACE/GPS/Topex/Jason
  - Degree-1 mass harmonic and geocenter motion
  - Accuracy improvement

• Secular Simultaneous Inversion Development: Viscoelastic Earth
  - Dynamic Approach: Secular present-day trend + GIA or PGR
  - Kinematic Approach: Present-day trend and PGR spherical harmonics
  - Secular geocenter motion and International Terrestrial Reference Frame
Publications (2006-2008) and Support


**Current Support**

NASA ESI GRACE Program: FY2008 - 2010, $160K per year

NASA IPY Program: FY2007 - 2009, $135K per year for Wu

NASA MeaSure Program: FY2008 – 2012, 0-0.2 FTE for Wu, PI: Zlotnicki

**Pending Support**

NASA ESI Program: FY2009 - 2011, $200K per year

NASA ESI Program: FY2009 - 2011, 0.25 FTE for Wu, PI: Gross
Surface Mass Variations and Data

\[ \sigma(R_e, \vartheta, \varphi, t) = \sum_{n=1}^{\infty} \sum_{m=0}^{n} \sum_{q=c,s} M_{nmq}(t)Y_{nmq}(\vartheta, \varphi) \]

Signatures

- Gravity change
- Surface displacements
- Ocean Bottom Pressure (OBP) change
- Geocenter motion, Earth rotation
- Relative Sea Level

Uncertainty Reduction

- GRACE + N=1 + OBP
- Unimodular Optimal Point and Regional Averages
- Full Data Covariance Matrix (Correlation)
- More Realistic Quantitative A priori
Significance of Geocenter Motion

Effects of 1 mm Geocenter Motion on GRACE Mean Mass Change Determination

<table>
<thead>
<tr>
<th>Geocenter</th>
<th>Eustatic Sea Level mm</th>
<th>Mean Antarctic Ice mm</th>
<th>Mean Greenland Ice mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>-0.46</td>
<td>0.07</td>
<td>1.26</td>
</tr>
<tr>
<td>Y</td>
<td>-0.24</td>
<td>0.49</td>
<td>-1.1</td>
</tr>
<tr>
<td>Z</td>
<td>-0.51</td>
<td>-5.6</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Satellite Tracking: \( \vec{S}_{cm} - \vec{S}_{cn} \)

Inverse Determination: \( G(\theta, \varphi, t) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} G_{nmq}(\theta, \varphi, p..) M_{nmq}(t) \)

Present-trend and GIA: \( \dot{G}(\theta, \varphi, t) = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} G_{nmq}(\theta, \varphi, p..) \dot{M}_{nmq}(t) + \dot{G}^\nu \)
Compare SLR, Climate Model and GPS/OBP Inversion

Imminent JGR Submission
- SLR from ITRF2005 Altamimi
- AOW Mass Model van Dam
- GPS/OBP Inversion
$M_{20}$ Surface Density from GPS/OBP, SLR, GRACE
Simultaneous Global Inversion for Present-Day Trend + PGR
Dynamic Approach

- Viscoelastic Earth Response
- Coupling of present-day trend and GIA signatures in modern data

\[ \dot{V}_{lm} = \dot{V}_{lm}(\dot{M}_{lm}^{\text{CUR}}) + \dot{V}_{lm}^{\text{GIA}}(M_{\text{past}}, \tau, \nu) \]

- Combination of data with different physical origin for separation
- Combination of modern and historical data for time resolution
- Adapted viscoelastic Earth model (with n=1) + sea level equation solver

Effects of Ice Model On GIA CM-CF

<table>
<thead>
<tr>
<th>Model</th>
<th>ICE-5G/ IJ2005 mm/yr</th>
<th>ICE-5G mm/yr</th>
</tr>
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<tbody>
<tr>
<td>Axis</td>
<td></td>
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</tr>
<tr>
<td>X</td>
<td>-0.12</td>
<td>-0.08</td>
</tr>
<tr>
<td>Y</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>Z</td>
<td>-0.49</td>
<td>-0.26</td>
</tr>
</tbody>
</table>

Geocenter Velocity (mm/yr)

- Lower Mantle Viscosity (Pa s)
Simultaneous Global Inversion for Present-Day Trend + PGR
Kinematic Approach

- **Objectives**
  - Improve PGR and Present-day trend estimation with GRACE/GPS/OBP
  - Separate estimation of geocenter velocities due to present trend and GIA

- **Data Equations:**

  **Geocentric Velocity:**

  \[
  \dot{s} = \sum_{n=1}^{\infty} \sum_{mq} \left( (a_n \dot{M}_{nmq} + b_n \dot{M}^{\nu,k}) Y_{nmq} \hat{e}_r + (c_n \dot{M}_{nmq} + d_n \dot{M}^{\nu,l}) \left( \partial_{\theta} Y_{nmq} \hat{e}_\theta + l'_n \frac{1}{\sin \theta} \partial_{\phi} Y_{nmq} \hat{e}_\phi \right) \right) \]

  \[- \frac{4\pi}{\sqrt{3}} \frac{a^3}{M_E} \left( \dot{M}_{11c} \hat{e}_x + \dot{M}_{11s} \hat{e}_y + \dot{M}_{10c} \hat{e}_z \right) + \vec{\omega} \times \vec{X}_i \]

  **Ocean Bottom Pressure:** \[ \dot{P} = gO(\theta,\phi) \dot{M} \]

  **Geoid:** \[ \dot{N} = \sum_{n=2}^{\infty} \sum_{mq} \left( f_n \dot{M}_{nmq} + g_n \dot{M}^{\nu,k} \right) Y_{nmq} \]

- **Method to Separate Geocenter Velocities due to Present-Trend and GIA**
Secular Geocenter Motion and International Terrestrial Reference Frame Stability

- Geocenter CM = Datum of Satellite Tracking = origin of the ITRF
  - What is the geocenter motion rate? 2 mm/yr in Z?
  - How accurate is the rate determined? ±2 mm/yr?
  - Not acceptable since mean sea level only rises 2-3 mm/yr!

- Major Problems of Current ITRF Realization
  - Secular Frame, not instantaneous – TRF needs to be specified at all times
  - Different techniques are not combined weekly – enormous loss of info

- New Method for ITRF Realization
  - SLR/VLBI/GPS/DORIS data weekly combination
  - Co-located sites moving together
  - Effects of non-secular motions